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A SPECTROSCOPIC STUDY OF THE SPARK SPECTRUM

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In the past decade much study has been given to the changes produced in the spark spectrum by varying some of the quantities connected with the spark or its circuit. This work, however, like the earlier researches, consists of many independent investigations dealing with special phases of the subject and conducted with quite dissimilar apparatus. Naturally, different results were obtained and conflicting conclusions reached. Thus one investigator concluded that the main factor in determining the appearance of the spectrum is pressure;[1] another, vapor density;[2] another, temperature;[3] another, nature of discharge;[4] another, inductance;[5] etc. More recent investigators[6] consider pressure, capacity, and inductance to be important factors; but have varied them, with the exception of pressure,[7] through small ranges. The observed displace-

[1] See footnote 4, p. 222. Cailletet, *Comptes Rendus*, **74**, 1282. 1872.

[2] Cazin, *ibid;* **84**, 1151, 1877; see also *Phil. Mag.*, **3**, 153, 1877; Monckhoven, *C. R.*, **95**, 378, 1882.

[3] Plücker and Hittorf, *Phil. Trans.*, **155**, 1, 1865; Salet, *Ann. chim. et physique* **28**, 5, 1873; Secchi, *C. R.*, **70**, 79, 1870.

[4] Stearn and Lee, *Proc. Phil. Soc. Liverpool*, **28**, 1874; also *Phil. Mag.*, **46**, 406, 1873.

[5] Hemsalech, *Journal de physique*, **9**, 437, 1900.

[6] Hale, *Astrophysical Journal*, **15**, 132, 1902; Hale and Kent, *Ibid.*, **17**, 154, 1903; Kent, *ibid.*, **17**, 286; Mohler, *ibid.*, **10**, 202, 1899.

[7] Hale and Kent (*loc. cit.*) took spectra at pressures up to 53 atmospheres.

ments of the spectral lines of certain new stars, and Wilsing's[1] idea that these displacements might be due to very high pressure[2] in the stellar atmosphere, gave the subject added interest.

The present work was undertaken in the hope that some of the questions involved could be answered, and the facts already known correlated, by a study of the effect produced upon the spectrum by separately varying through wide ranges each of the quantities[3] connected with the spark or its circuit.

Work upon the spark spectrum may be divided into two parts: (1) the study of change in the width and character of its lines, as well as other changes in its general appearance, and (2) the study of shift, or change of wave-length of its lines. Recent investigations in which spectroscopic apparatus of high dispersive power was used have been mainly concerned with shift, and have discussed rather incidentally the character of the spectrum. The present research deals with both, but principally with the character of the spectrum and its relation to the character of the spark.

The ultimate cause of the above changes in the spectrum cannot consist in changes of capacity, pressure, etc., *per se*, but in resulting changes in the light-emitting vapor. In all probability the appearance of the metallic spectrum depends solely upon the amount, density, pressure, nature, and temperature of the metallic vapor in the spark; and the capacity,[4] pressure, etc., are of importance only in so far as they affect these conditions of the vapor. The quantities (capacity, etc.) have been varied through the ranges given in Table I, and it has been found that the changes thereby produced in the spectrum may be explained from the above standpoint. Thus the relation between the separate effects of these quantities is shown. It was thought that the potential-drop over

[1] *Astrophysical Journal*, 10, 113, 1899.

[2] Humphreys and Mohler had previously shown (this *Journal*, Vols. 4, 5, and 6, 1896–97) that when the arc is surrounded by a gas at high pressure, its spectral lines are given slight displacement (shift) toward the red relatively to those of the normal arc.

[3] See table, p. 223.

[4] In all work at *pressures* above one atmosphere, the *spark* whose spectrum is being studied takes place between insulated *electrodes* in a *spark chamber* filled with a *gas* under pressure. In this spark circuit there is usually a certain *capacity*, *inductance*, and *resistance*. In this paper the above italicized words will always be used in the above sense, unless otherwise stated.

the spark, the current through it, and the energy developed in it, as well as the resistance of the spark circuit (quantities not previously measured in such work), might be closely related to the character of the spectrum. Accordingly these, too, were investigated.

Since much of the apparatus and all the instruments were made especially for use in the work, a somewhat detailed account of these will precede the discussion of results.

TABLE I

Quantity	Range	Ratio. (Of Max. to Min.)
Pressure	1 to 100 atmospheres.	100
Capacity	0.00145 to 0.2231 microfarads.	150
Inductance	0.0135 to 140 milhenrys.	10,400
Potential-drop	2000 ? to 12,000 volts.	6 ?
Nature of gas	Air, Hydrogen, CO_2.	
Energy in secondary	4, or less, to 125 watts.	30
Resistance	0.2 ohms plus spark to 78 ohms, plus spark.	100 ?
Exposure time	8 seconds to 10 minutes.	75
Oscillation period	1.03×10^{-6} to 1.1×10^{-3} seconds.	1000

APPARATUS

Arrangement.—The arrangement of the apparatus as used is shown diagrammatically in Fig. 1. *T* is the transformer; *C*, the

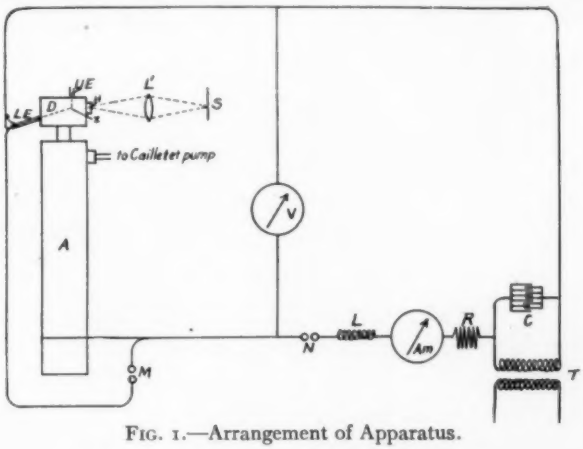


FIG. 1.—Arrangement of Apparatus.

capacity; *L*, the inductance; *R*, the non-inductive resistance (used in only a few cases); *Am.*, the hot-wire ammeter used to measure

the current through the spark Z that takes place between the carefully insulated electrode *L. E.* and the adjustable uninsulated electrode *U. E.;* V, the static voltmeter[1] for measuring the potential-drop over the spark Z. The light from this spark, which may be called the working spark, passes through the heavy glass window H of the steel compression chamber D, and is concentrated by the lens L' upon the slit S of a Rowland concave grating[2] spectroscope. The adjustable spark-gap N is sometimes used to maintain the disruptive character of the discharge. A similar spark-gap M, in parallel with the working spark, serves to protect the insulation of the lower electrode *L. E.* from excessive potential differences. R, L, and C can each be varied through wide ranges or thrown out of the circuit entirely.

Compression tube and compression chamber.—To avoid being limited to the pressures obtainable from ordinary commercial gas cylinders, a special form of tube A and attached compression chamber D was made as shown in Fig. 2. As water from the Cailletet pump is forced into tube A at s, mercury is driven from the annular space C into tube B, thereby forcing the gas from B through u (controlled by valve a) into the spark chamber D. If still higher pressure in D is desired, the valve a may be closed and B may be refilled with gas (usually at about 10 atmospheres pressure) through orifice b, which is controlled by a valve c (not shown) similar to a. This additional gas is then also forced into D as before.

The glass window H, protected by a lead washer on each side, is tightly held on D by the screw-cap I. Lead washers are used at all joints not exposed to mercury. For other joints leather or rubber washers are used.

When hydrogen was used it was generated with an ordinary *Zn–HCL* generator and collected in a large tank, from which it was pumped through drying-tubes into B. When CO_2 was used it was obtained from a commercial compressed gas cylinder.

Pin valves, consisting of a steel cone screwing down into an accurately ground steel seat, and packed with rubber as shown, were found very serviceable at a and at c (not shown). The elec-

[1] Kelvin vertical quadrant type calibrated from 4,000 to 17,000 volts.

[2] 14,438 lines per inch, and 21 feet radius of curvature.

trodes *U. E.* and *L. E.* were likewise packed with rubber to prevent leakage. Nevertheless, considerable trouble with leakage was

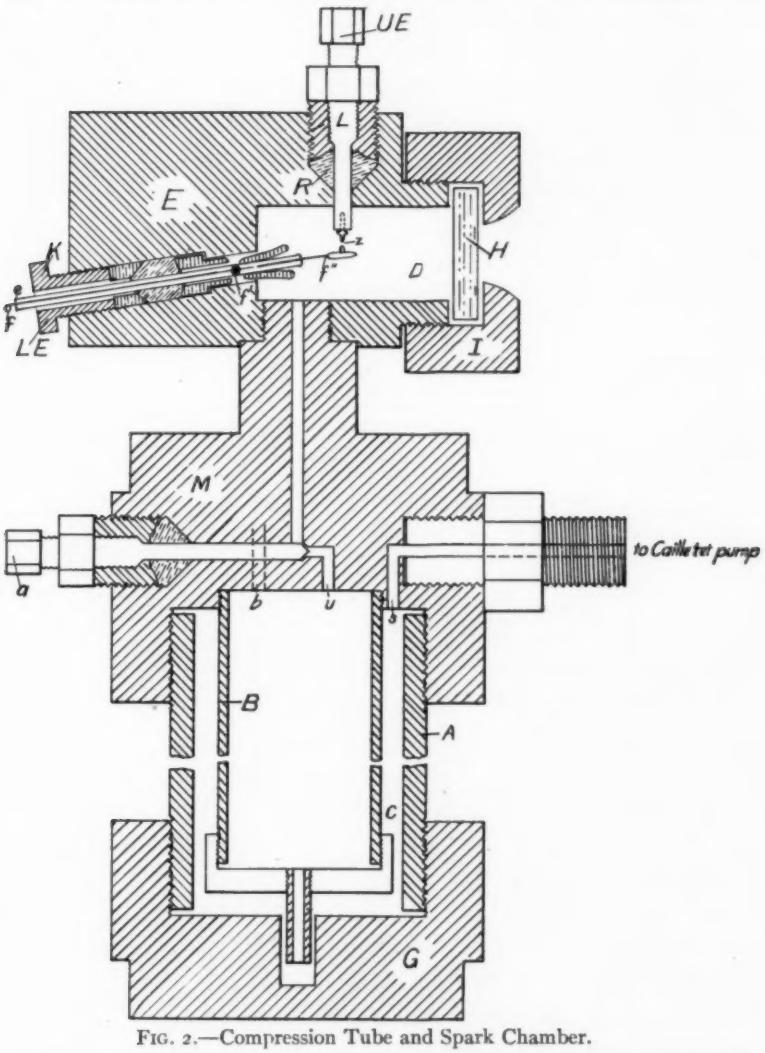


FIG. 2.—Compression Tube and Spark Chamber.

experienced at pressures from 80 to 100 atmospheres, especially with hydrogen.

The electrodes.—The upper electrode, *U. E.*, which is in electrical contact with the compression chamber, can be screwed up or down

to alter the length of the spark-gap and consequently the potential fall over the spark. With larger capacity, and at high pressures, the spark-gap must be very short, or the spark will either not pass at all or else it will pass through the parallel spark-gap *M* (Fig. 1), even though the latter be 6 to 10 mm in length. The intense heat developed under these circumstances readily fuses the electrodes together, while a slight turn of *U. E.* usually separates them so far that no spark passes; thus continual adjustment is necessary. For this reason, what might be termed an hour's exposure really consists of several thousand exposures varying from one-twentieth to one-half second. At lower pressures, say 50 atmospheres or so, the spark often runs steadily for several minutes.

For the lower electrode *L. E.* various forms were tried, and many glass tubes were exploded before one was made that could long withstand the great heat, and the electrical and mechanical stresses. In the final form (*L. E.*, Fig. 2) a copper wire *j* has a short piece of platinum wire tightly fitted into a hole bored in one end. To this platinum wire is fused a small globule of fusible glass, which is then inserted into the tube *e*, and sealed to it at *j'* as shown. With this method of sealing, it will be seen that the gas pressure on the tube is from without, and hence easily withstood. To prevent the wire *j* from locally heating and thereby fracturing the tube *e*, mercury is put in around the wire and kept there by a vulcanite cap (not shown) containing soft wax. This cap is connected with the nut *K* by a vulcanite clamp (not shown), and thus prevents *e* from being forced out by the pressure in *D*. The wire *j''*, making electrical contact with the platinum wire, and hence with *j*, through a drop of mercury in *e*, completes the lower electrode. (*K* to $e=3$ in.)

The transformer T (Fig. 1) has its primary connected with the city circuit of 108 volts and 60 cycles per second. The secondary gives a potential of 15,000 to 20,000 volts, and can furnish a large current, which is necessary when large capacity is used.

The condenser consists of thin (No. 33) sheet brass 9×18 in., shellacked to glass plates 12×22 in. Eighty of these are arranged on edge in an oil tank filled with kerosene of high flashing-point. The brass sheets, properly grouped, are connected with the mercury cups of a switch-board by which the various groups may be con-

nected in parallel or series, and the capacity varied from 0.00145 to 0.2231 microfarads. This condenser is efficient and serviceable, and was built after it was found that the heavy discharges and continuous use burned up the tinfoil and melted the paraffin of a tinfoil-on-glass condenser incased in paraffin.

The variable inductance is composed mainly of 2,000 ft. of No. 14 rubber-covered wire. By connecting to different terminals, inductances from 0.14 to 140 milhenrys are obtained. For smaller inductances special coils were used.

Non-inductive resistance.—Since resistance causes a damping of the oscillations, and may even altogether prevent them, it seemed important to study its effect upon the spectrum. To avoid simultaneously introducing considerable inductance, this resistance is made nearly non-inductive by using high resistance (IA) wire which is wound back and forth over two rows of pegs about 4 inches apart. Computation gives 0.03 milhenrys for the 78 ohms resistance.

The ammeter.—In the hot-wire ammeter shown in Fig. 3 a strip of sheet brass, *D*, 0.5×5×500 mm, stretched between the binding

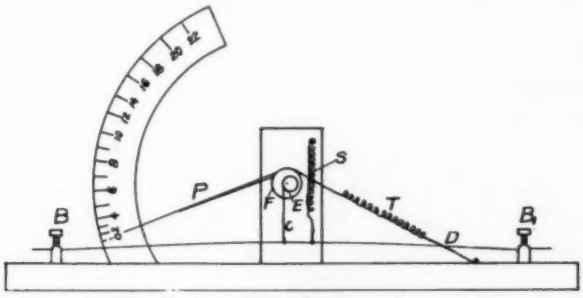


FIG. 3.—Hot-wire Ammeter.

posts B, B_1, carries the current. As the current heats this strip it elongates, and the spring s contracts, thus permitting spring T to rotate F and thereby raise the pointer P, at the same time winding the thread C upon the eccentrically placed pulley E. E was so placed as to give a fairly uniform scale, as shown. This instrument was calibrated from 2 to 22 amperes, while a similar one with a smaller conductor was used for currents from 0.25 to 5 amperes.

The wattmeter.—Various devices were tried for ascertaining the amount of energy developed in the spark. In the first, immediately

after taking the spectrum, the spark, having all other conditions connected with it kept unchanged, was made to take place inside a special calorimeter. From the rate of temperature rise of the calorimeter, the watts developed by the spark could be computed. However, the spark as used occurred in a gas at high pressure (in the spark chamber), while in the calorimeter it occurred in air at one atmosphere pressure. This makes a great difference in the amount of energy developed, so this arrangement was abandoned.

The spark calorimeter was next used simultaneously with the working spark, and in series with it. This insured the same current in each spark; so that the energy in each was proportional to the potential drop across it. Uncertainty as to the measurement of these potentials was here a fatal objection.[1]

Finally the compression chamber itself was made to serve as spark calorimeter, and was calibrated as a wattmeter. For the calibration a spiral of wire heated by a direct current, and using a measured number of watts (say 16.5) was placed in the compression chamber, and temperature[2] readings were taken every minute. Curve *A*, Fig. 4, is obtained by plotting these temperature readings as ordinates and the corresponding times as abscissae. The other six curves were obtained in the same way. In all cases the calibration was begun with the chamber at room temperature; so this temperature was used as an arbitrary zero in plotting.

To illustrate the use of these curves, suppose in 20 minutes the spark were to heat the spark-chamber from room temperature to 12° above. The point whose abscissa is 20, and whose ordinate is 12, is about half-way between curves *B* (19 watts) and *C* (31.3 watts); hence about 25 watts would be the power expended in the spark.

Though the compression chamber radiates heat freely, and also imparts it to the compression tube by conduction, errors from these causes are eliminated because the same heat losses occurred during the calibration. Indeed, for fifteen or more watts, it seems to be a sufficiently sensitive and accurate wattmeter. The fact that the

[1] See multiple spark-gap, footnote 3, p. 231.

[2] The thermometer was inserted into a hole bored into the wall of the compression chamber and containing some mercury to make good thermal connection.

curves produced cut the x axis within 20 or 30 seconds of the origin, and also that after breaking the circuit (as at z, curve F) the indicated temperature reaches a maximum in less than a minute, show that the heat is quickly transmitted from the spark to the thermometer.

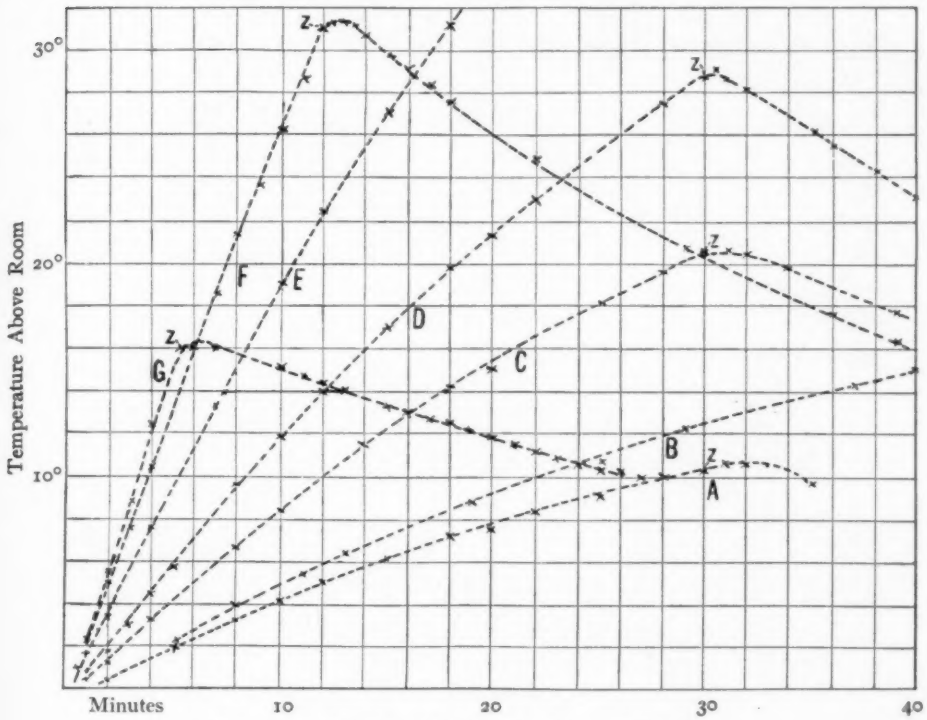


FIG. 4.—Calibration of Spark-Chamber as Wattmeter.

Z marks the point where the circuit was broken.
A=16.5 Watts; B=19; C=31.3; D=47; E=70.5; F=100; G=112 Watts.

The camera.—The camera, of the usual form for gratings, may be raised and lowered, by which motion, and the use of a suitable screen, a dozen or so spectra may be taken on the same plate (see No. 174 and footnote 2, p. 235). This method can be used in comparing the general appearance of the lines of successive spectra as some one of the quantities, capacity, inductance, etc., is varied. To determine shift, the method used by Humphreys and Mohler[1] was adopted.

[1] *Loc. cit.*, p. 222.

RESULTS

Electrical conditions in the spark.—The true value of the current through the spark, the watts used in it, and the potential-drop over it manifestly cannot be directly inferred from the observed values obtained by the ammeter, the wattmeter, and the voltmeter, respectively, because the current through the spark is flowing a very small fraction of the time. Though the spectrum varies much less with these observed values than with change of capacity or pressure, etc., it is believed that the actual values of the current and energy during the brief intervals that the current flows, are of prime importance, and that it is through changes in these values mainly that change of capacity, pressure, inductance, etc., affect the spectrum.

These instruments were calibrated either with direct current circuits or alternate current circuits of moderate frequency. The current and potential variations in the case of the spark are of very high frequency and are also exceedingly irregular; so that direct comparison between measurements of them and those of low frequency cannot be made without serious error.

As this is, to my knowledge, the first attempt[1] at measuring the current, the potential-drop, and the energy of the spark in such work, I shall endeavor to interpret the readings obtained by showing to what extent the various actual values[2] of the current, the potential, etc., contribute toward producing the observed deflections. This can be best shown by curves.

In Fig. 5, curve I is intended to represent the current in the secondary of the transformer; while curve II, it is believed, represents the potential-drop over the spark, which is practically the potential difference between the needle and the quadrants of the voltmeter. Although the condenser when fully charged produces an opposing *E. M. F.* of several thousand volts, the inductance of the secondary of the transformer (80,000 turns) is too great to permit of sudden variations of the current; so that curve I should be a nearly smooth sine curve as shown.

[1] Battelli and Magri (*Phil. Mag.*, **5**, 620, 1903) in an investigation of the oscillation period measured the potential and the energy of the spark in air at one atmosphere.

[2] Mean square values for various intervals in each cycle.

With curve II, suppose at the time represented by a_2, the condenser is completely discharged. The value of the current at that instant is aa', which rapidly charges the condenser to a potential $b_2b'_2$, whereupon the spark-gap breaks down and a group of oscillations follows. For several oscillations the potential-drop over the condenser is but little less than $b_2b'_2$, but the drop over the spark is a small part of this (as shown), because the inductance of the spark is very small compared with the total inductance, and because

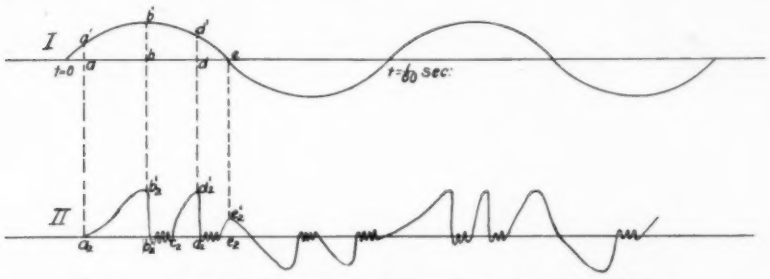


FIG. 5.—Curve I: Current in the Secondary of the Transformer.
Curve II: Corresponding Potential-drop over Spark.

its resistance during oscillations also is probably very small (p. 233). These oscillations continue from b_2 to c_2,[1] after which the above cycle is repeated.[2] At e_2 the condenser is not charged to the sparking potential, and, as the current reverses at e (curve I), no spark occurs until it is charged in the reverse sense to the potential $f'^2f'_2$. From the curve it is evident how insignificant[3] a part the potential

[1] The distance b_2c_2 is greatly exaggerated in the curve. From photographs of the oscillations obtained by the use of the rotating-mirror method and from other calculations, it seems that ordinarily the oscillations last about 1/100 of the time; i. e., $b_2c_2=1/100\ b_2d_2$, roughly.

[2] Here the special case has been taken in which the secondary of the transformer furnishes in one-half cycle (1/120 sec.) enough electricity to charge the condenser twice to the potential required to break down the spark-gap. From the photographs of the spark taken with a rapidly moving plate this number was found to vary from one to twelve or more and to be different even for successive half-cycles.

[3] In some allied work seven brass knobs, placed in a row and separated by millimeter spaces, were used as a spark-gap. It was found that the potential over four of these sparks (exclusive of the two end ones), instead of being 4/6 of the total, was quite too small to be measured or even detected. Doubtless this was due to the fact that only from b_2 to c_2 (i. e,. during oscillations) was there any potential-drop over the voltmeter, except through slight charging by the streamers that precede the spark.

during the interval b_2c_2 contributes to the resulting deflections of the volt-meter needle, and hence how hopelessly masked its value is.

It is just at this time, too, when the current is surging through the spark, that the potential-drop over it might be most expected to affect the spectrum. The potential read is probably about two-thirds the discharge potential[1] $b_2b'_2$ (see curve II).

This discharge potential may affect the character of the light in three ways. First, through disintegration of the electrodes by the enormous potential gradient,[2] particles may be thrown into the spark and by their momentary incandescence form the continuous spectrum. Second, through the effect the initial electrostatic stresses may have upon the light-vibrations, if they result from direct electrical action and not from heat. My results throw no light upon this question, but I simply suggest the possibility of such effects.

The third way, through its effect upon the initial discharge, is discussed on p. 248.

The hot-wire ammeter (Fig. 3) integrates the instantaneous values of C^2R, C being the current through the brass strip D, and R its resistance. Evidently the current is practically zero except during oscillations, i. e., from b_2 to c_2, curve II; so that during these brief intervals, which probably constitute about 1/100 of the time, the current must be very strong to produce as great heating effects as were observed.

Some computations as to the probable current, potential-drop, watts, and resistance of the spark when taking spectrum No. 71*c* will be given. This particular case is chosen because many other spectra were taken under conditions not greatly different. The indicated current was 20 amperes, voltage, 7,000. (See No. 71 *c*, Table II.) To produce this heating effect with an intermittent current flowing 1/100 of the time requires of course $\sqrt{100}\times 20$ or

[1] I find this estimate in accord with the discharge potentials given by Battelli and Magri (*loc. cit.*, p. 230) for various spark-lengths. Of necessity, since their work was at one atmosphere, I compare the length of their spark ("explosive distance") with that of the parallel spark *M*, (Fig. 1), which was so adjusted that the spark occasionally took that path.

[2] Judging from the slight motion (p. 226) of the electrodes required to short-circuit the spark at pressures from 80 to 100 atmospheres, it seemed that the spark-gap was less than 0.05 millimeter. With 10,000 volts over the spark, the potential gradient would then be more than 2×10^6 volts per centimeter.

200 amperes.[1] Again from the capacity, voltage, and frequency, we obtain by computation (as on p. 243) 218 amperes.

Both of these values of the current are necessarily rough approximations because of the uncertainty (1) as to the fraction of the time that the oscillations exist, and (2) as to the value of the charge on the condenser during these oscillations. The current, however, is very probably between 100 and 200 amperes in many cases.

The wattmeter (calorimetric—see p. 228) probably gives the average value of the watts within 10 per cent. More energy is radiated through the glass window of the compression chamber by the spark than by the hot wire with which the calibration was made, but this is a small fraction of the energy in either case, so that the error introduced is slight.

When taking No. 71 (see above), the wattmeter indicated about 100 watts. Assuming the oscillations to persist 1/100 of the time, the average value of the watts during oscillations is 10,000, which with 200 amperes would require 50 volts drop over the spark, showing the resistance of the spark to be 0.25 ohm. This is a very low resistance,[2] but the spark is exceedingly short, and the great amount of energy in it must develop considerable metallic vapor.

The aim has been to vary, one at a time as far as possible, the quantities given in Table I, and study the changes thereby produced in the spectrum. The capacity, inductance, resistance, and pressure may be varied independently; but the current and energy depend very much upon these values and upon the amount of energy furnished to the primary. Indeed, I believe the effect produced upon the spectrum by varying the capacity, inductance, and resistance is due almost entirely to the resulting change in the energy of the spark, which in turn determines the amount of metallic vapor present. We may then consider pressure, capacity, inductance,

[1] This is without taking into account the increase of R due to "skin effect" (see Drude's *Physik des Aethers*, p. 245). Adapting the formula (Rayleigh's $R = R_1 \pi r \sqrt{\frac{\mu\sigma}{\tau}}$) which is designed for round conductors, to apply to the brass strip (D) of the hot-wire ammeter (p. 227), I find that with the oscillation period used for No. 71, the skin effect just about doubles the resistance. This doubles the heat effect, so instead of 200 amperes it would require but $200/\sqrt{2}$ or 140 amperes.

[2] Battelli and Magri, *loc. cit.*, p. 230, obtain less than an ohm for very much longer sparks (at one atmosphere pressure).

resistance, potential-drop (i. e., length of spark) as the independent variables; and current, energy, and oscillation period, as the dependent variables. The energy and also the current may, however, be varied by varying the energy in the primary of the transformer (see No. 91, Table II), leaving the above independent variables unchanged.

In studying the effect of each quantity, fifteen or twenty spectra have, as a rule, been taken; so that it is believed that errors arising

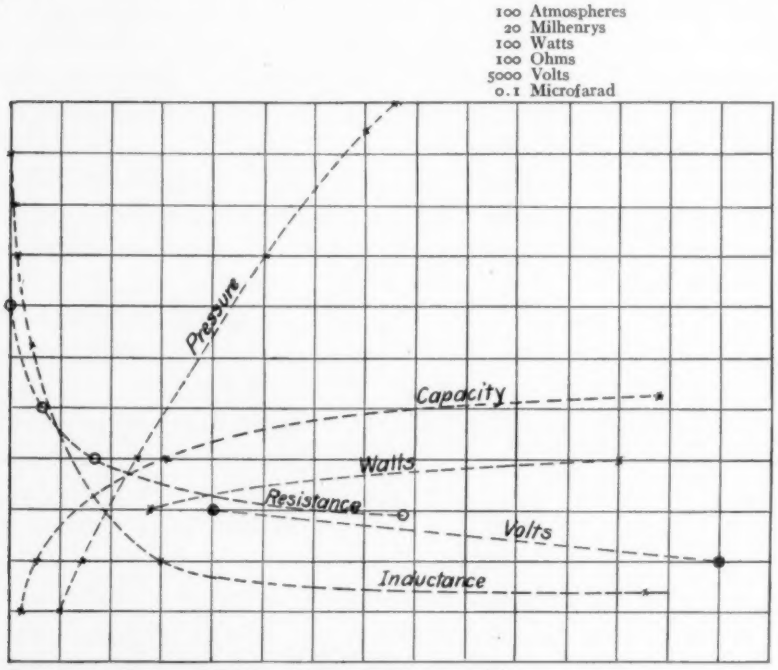


FIG. 6.—Ordinates: Widths of Lines (arbitrary scale).
Abscissae: Pressure, Capacity, Inductance, etc. (on different scales).

from accidental variations have been avoided. In Fig. 6, several curves have been plotted having relative width of reversed lines as ordinates and the corresponding values of the given variable for abscissae. The ordinates are plotted to a different (arbitrary) scale for each curve, since in separately studying the effect of each variable, it is obvious that nothing would be gained by having the scale uniform. The relative widths are averages based upon estimates from several

lines in each case. Frequent references to these curves will be made in the following pages.

Effect of pressure.—The general effect of increase of pressure[1] is to cause the bright lines to increase in width, then reverse, and finally disappear entirely, leaving only, or mainly, reversed lines on a strong continuous spectrum. This is shown, for example, in No. 170,[2] in which the pressure (see pressure-curve, Fig. 6) is varied as shown on the margin, while the capacity, inductance, etc., were kept constant (see Table II). This effect has the same trend whether the gas used is carbon dioxide (No. 170), air (No. 48), or hydrogen (No. 85); and also whatever value the capacity, inductance, etc., may have, provided these values be kept constant for the series.

If the capacity is small and the inductance large, the pressure must be much higher to produce as wide lines as if the reverse were true; thus No. 120 *a* (70 atmospheres) has not so wide absorption lines as No. 103 *d* (20 atmospheres), Nevertheless, the pairs Nos. 112*a* and 120*a*, or Nos. 112*b* and 120*b*, or Nos. 112*c* and 120*c*, show that in this case also increase of pressure widens the lines if the capacity and inductance are kept constant. These few spectra demonstrate the necessity for stating all the conditions under which such spectra are taken; and also show how difficult it is to draw any conclusions from many previous researches in which this fact has been overlooked.

In case the lines of the different spectra on a plate—e. g., *a*, *b*, *c* and *d* of No. 170—appear slightly displaced relatively to each other, it must not be construed as shift (i. e., change of wave-length), since it is probably due simply to motion of the camera. To determine shift great care was taken not to jar the camera, and a different method was employed (see pp. 229 and 249).

Effect of capacity.—Increasing the capacity has very much the same effect upon the spectrum as increasing the pressure, as shown

[1] Compare work by Hale and Kent, *loc. cit.*

[2] Hereafter the spectra will be referred to simply by number, as No. 170*a*, No. 170*b*, etc. The plates, together with the conditions under which they were taken, are given in numerical order in Table II, which also shows, in column 2, where the photographic reproductions may be found, if given.

by No. 112, No. 116, and No. 120, in which capacity[1] alone is varied. No. 112*a* is essentially a bright-line spectrum with but one line (λ 3720.08) reversed; while No. 112*d*, with 60 times as much capacity, has many and very wide reversals—in fact, but two lines that are not reversed. These are λλ 3709 and 3765, the middle lines of two similar and interesting triplets. In the latter triplets I find changes produced by capacity and pressure similar to those observed by Hale[2] with the spark under water. The changes of intensity of the different lines of such a triplet relatively to each other is easily noticed because of the close proximity of the lines; so that they are among the best lines to observe throughout the various spectra.

Naming the lines of the former triplet, *a*, *β*, and *γ* (*a* being λ 3706), their relative intensities may be expressed by 5, 1, and 7, for the arc No. 149*a*); by about the same numbers for No. 174*k* (spark in CO_2 at 1 atmosphere); by 8, 1, and 5, for No. 112*a*; by 5, 1, and 5, for No. 112*b;* and by 1, 1, and 5, for No. 112*c*. *β* is not reversed in No. 112*d*. *γ* is, but also shows faintly the bright emission line on each side of the reversal, while *a* shows no trace of emission line. The intensity of a bright line diminishes very rapidly with increase of capacity (or any other cause of reversal), when the capacity is such as to produce barely perceptible reversal of the line, because the reversal is just at that part of the emission line that would otherwise be most intense. This accounts for the above-cited rapid diminution in intensity of *a*, which reverses more readily with increase of capacity than does *β* or *γ*. The increase in intensity of the middle line relatively to the outer two of the triplet observed by Hale may be explained in the same way. For this middle line (λ 3765.69) has just been shown to be one of the most difficult to reverse in that region of the spectrum. In No. 55, No. 75, and many others *a*, *β*, and *γ* are all reversed, and their relative intensities are about 6, 1, and 4, respectively; while in the solar spectrum (No. 199) they are 2, 1, and 3.

[1] The capacity-curve (Fig. 6) shows the relation between width of lines and capacity change under certain other constant conditions (inductance 140 milhenrys, pressure 50 atmospheres, etc.), and might have a somewhat different form under other conditions. The same may also be said of the other curves.

[2] *Loc. cit.*

PLATE IV

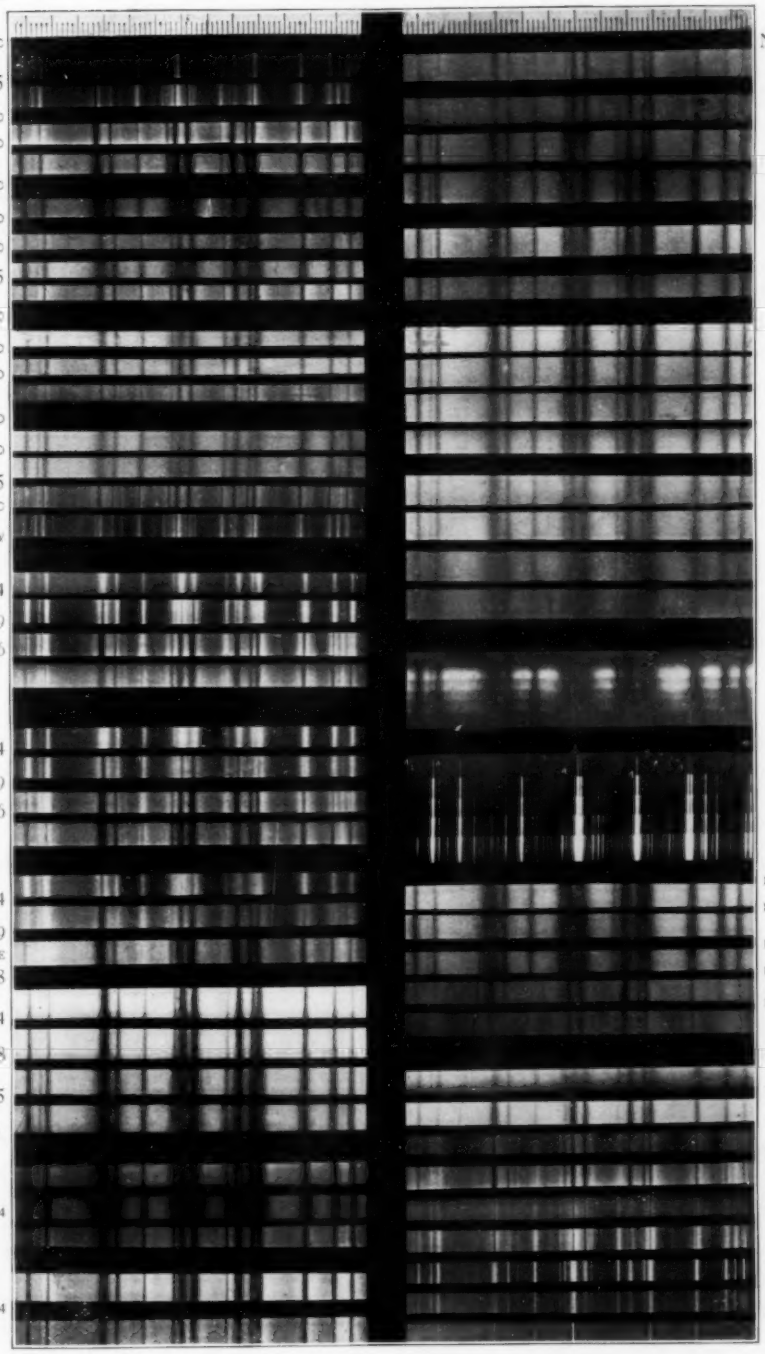


Plate	Pres.
No. 85*a*	10
85*b*	15
85*c*	30
85*d*	50
85*e*	70
79*d*	30
79*e*	60
79*j*	25
48*a*	60
48*b*	40
48*c*	20
170*a*	50
170*b*	40
170*d*	15
170*e*	10

Capacity	
116*a*	0.002
116*b*	.0054
116*c*	.0309
116*d*	.1286
112*a*	.002
112*b*	.0054
112*c*	.0309
112*d*	.1286
120*a*	.002
120*b*	.0054
120*c*	.0309

Inductance	
55*a*	0.868
55*b*	.594
55*c*	.278
55*d*	.155

Pr.	Ind.
103*a* 50	25.
103*b* 50	6.5
103*c* 20	0.014
95*a* 40	25.
95*d* 20	0.014

Plate	Res.
No. 63*a*	78
63*b*	16.9
63*c*	4.69
63*d*	0.25?

Watts	
91*a*	27
91*b*	120

Volts	
71*a*	2000?
71*b*	4100
71*c*	7000
71*d*	2000?

Exp. Time	
75*a*	600
75*b*	120
75*c*	20
75*d*	5
167*a*	180
167*b*	66
167*c*	26
167*d*	6
167*e*	2

Iron Arc	
2*a*	1
2*b*	6
2*c*	30
2*d*	120
2*e*	600

	Ind.	Cap.
161*a*	75	0.2231
161*b*	23	.073
161*c*	8.8	.073
161*d*	0.7	.0054
161*e*	0.7	.002

	Pres.	Gas
174*a*	50	CO_2
174*b*	50	*H*
174*c*	25	CO_2
174*d*	25	*H*
174*e*	15	CO_2
174*j*	15	*H*
174*g*	10	CO_2
174*h*	10	*H*
174*i*	5	CO_2

Effect of inductance.—Increase of inductance renders the discharge less violent and produces an effect upon the spectrum directly opposed to that due to increase of capacity or pressure. The reason for this will be discussed later (p. 248). Small variations do not produce a very marked effect (see No. 55); but No. 71*c* (0.0135 milhenry), and No. 112*c* (140 milhenrys) are radically different, the latter having several bright lines, the former none. Compare also No. 186 with No. 187.

Effect of resistance.—Resistance affects the spectrum[1] in the same way that inductance does, as shown by No. 63.

Energy in the spark.—The effect of increasing the energy in the primary from 120 to 720 watts (capacity, etc., kept unchanged), and thereby increasing the energy in the spark from 27 to 120 watts (see No. 91), is similar to that produced by increase of capacity, the absorption lines becoming, in this case, about one-third wider, and the continuous spectrum more intense. This effect is less than was expected (see p. 247).

Effect of potential.—As the potential is increased, by lengthening the spark-gap (see Fig. 2), the lines become slightly narrower (see No. 71), though just the opposite effect[2] might be expected (p. 248). The lowest potential used in this case is lower than that for which the voltmeter was calibrated, and is therefore uncertain; but this affects the result only in a quantitative way.

Nature of the gas.—In No. 149 several spectra were taken, using alternately air and hydrogen in the spark-chamber at the pressures shown, in order to compare the effects of these two gases.

A similar series (No. 174) was taken to compare carbon dioxide with hydrogen and hence with air. The pair *a* and *b* cannot be directly compared because *b* was much more exposed[3] than *a*, and consequently has narrower lines.

Equal exposure for such a series of spectra is very desirable,

[1] See resistance and inductance curves, p. 234. It will be observed that both these curves are roughly of the exponential type, $y=e^{-x}+b$, in which b is the limiting (minimum) width approached as inductance or resistance, x, is increased. The capacity-curve is of the form $y=a-e^{-x}$, in which a is a limiting (maximum) width of line.

[2] Perhaps the opposite effect could be obtained under other conditions of pressure, capacity, etc.

[3] See effect of exposure, p. 238.

but not easily attained, because different amounts of "smoke" on the compression-chamber window, and various other conditions, cause the proper time of exposure to vary from a few minutes to several hours. A small test-plate, slipped into an attachment at one side of the camera, which could be taken out and developed when it was thought the main plate was sufficiently exposed, was found very useful, but did not completely solve the problem of equal exposures. From these plates, however, from No. 55*a* and No. 186, and from plates taken for other purposes, I estimate the relative widths of the reversed lines to be 4, 5, and 6, respectively, for air, carbon dioxide, and hydrogen. The spark runs more steadily in hydrogen than in air or in carbon dioxide, which would probably cause more vapor to be produced and hence wider lines.

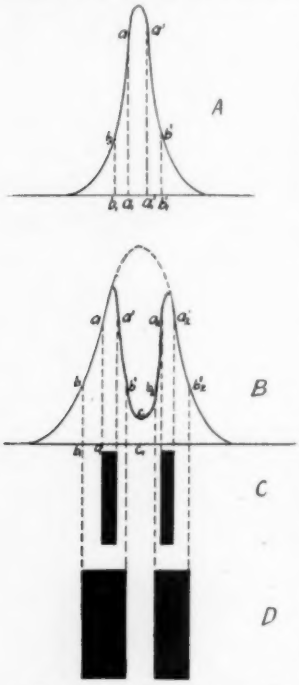


FIG. 7.—Intensity-curve for spectral line.

Duration of exposure.—In some previous work it was observed that the width of the bright lines increased markedly with the time of exposure (No. 2); and it was thought that for the same reason the reversed lines should be narrowed by increased exposure. This is shown to be the case by No. 167 and No. 75. No. 75*c* (20 sec.) has nearly twice as wide lines as No. 75*a* (10 min.).

The reason for this change in width is evident from a consideration of the intensity-curve in the immediate vicinity of a bright line[1] (*A*, Fig. 7), as given by Rayleigh[2] and Michelson[3]. If the time of exposure is so brief that intensities of less value than a_1a fail to noticeably darken the negative, a line of width aa' results; whereas a longer

[1] For a different application of the same idea see Kayser's *Spectroscopie*, II, p. 297.

[2] *Ibid.*, p. 343; *Phil. Mag.*, **27**, 298, 1889.

[3] *Astrophysical Journal*, **2**, p. 251, 1895.

exposure, which enables intensity b_1b to darken the negative, gives a line of width bb'.

Similar reasoning will show that if *B*, Fig. 7, represents the intensity curve[1] for a bright line partly reversed, then the short exposure will give a bright line of width a_1a_{11} (aa'_2), having a reversal of width $a'a_2$ as shown by *C;* while the longer exposure gives the line shown at *D*.

Order of reversal.—In general it may be said that certain lines reverse more readily[2] than others, and that any series of changes that increases the number and width of the reversals, will do so in approximately the same order, whether it be through increase of capacity or pressure, or through decrease of inductance or resistance. There are, however, cases in which the order of reversal seems to differ for some of the weaker lines. For example, No. 143*a* and No. 170*e* (Table VI) have each twenty-seven reversals, but not of the same lines throughout. Duration of exposure might cause such differences; for if certain lines have a relatively high value c_1c (*B*, Fig. 7), it will be clear that such intensity will suffice to darken the plate with long exposure so that no reversal occurs, even though with short exposures they do occur.

Increase of capacity and pressure tends to reverse more lines and widen those already reversed, while increase of inductance tends to narrow and also to suppress reversals. It appears, however, that capacity and pressure affect most strongly the prominent, easily reversed lines, and inductance, those difficult to reverse. Thus when both capacity and inductance (No. 134 and No. 120*c*), or pressure and inductance (No. 127), or all three (No. 137), are large, the strong lines have very wide reversals, while some of the weaker lines e. g., λ 3765, do not reverse at all. Compare these with No. 186 and No. 103*d* of low pressure and small inductance. No. 137 and No. 103*d* show the most striking difference. In No. 103*d*, λ 3765 shows as a reversed line and is not reversed in No. 137, although in the latter λ 3720 is three times as wide as in the former. I cannot explain this peculiar behavior of the lines. Inductance

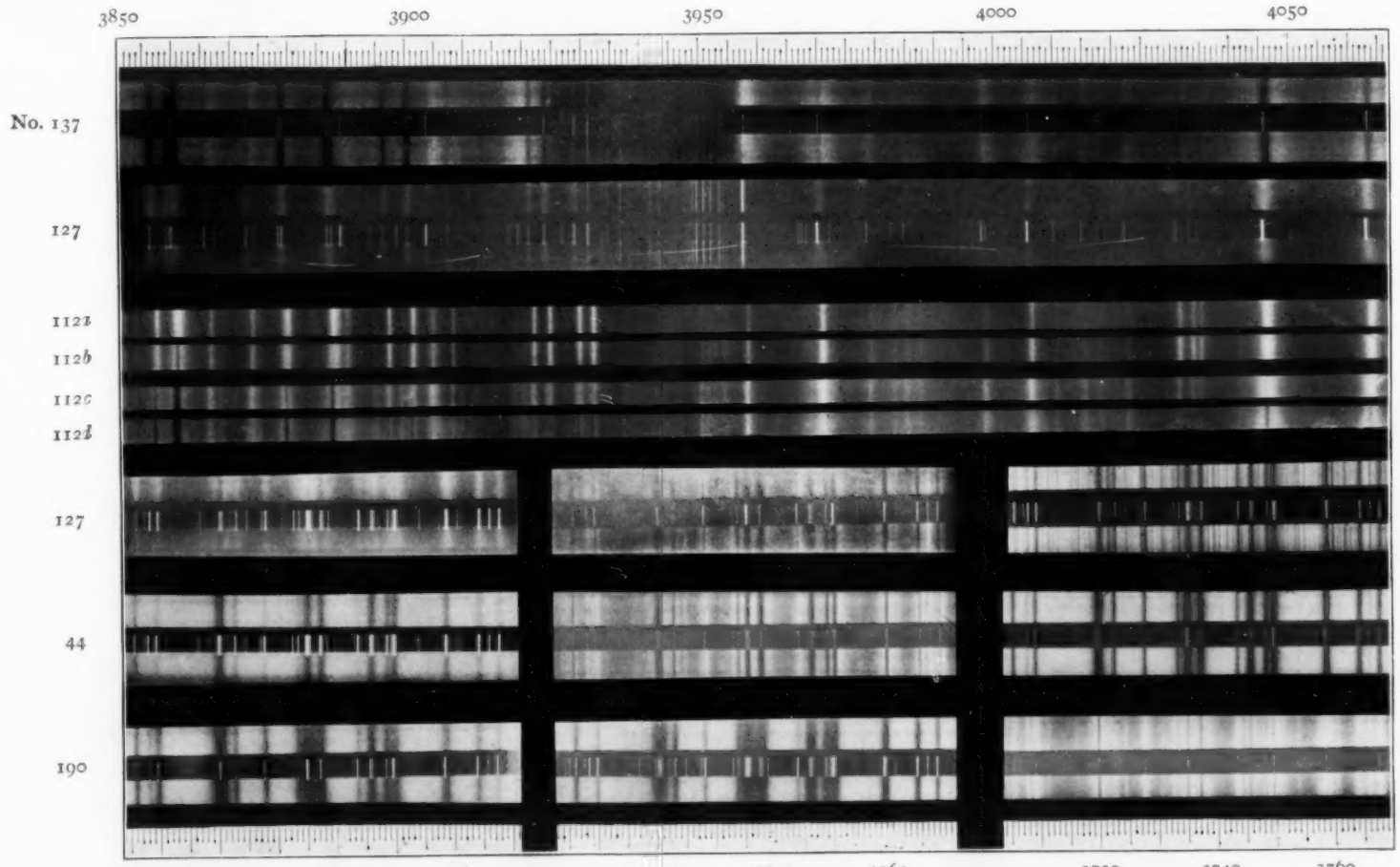
[1] See curve by Jewell, *Astrophysical Journal*, 3, 89, 1896; also Kayser's *Spectroscopie*, II, p. 360.

[2] Kayser's *Spectroscopie*, II, pp. 518, 296.

TABLE II

Spectrum No.	Plate No.	Pressure in Atmospheres	Inductance in Millihenrys	Capacity in Mfds.	Secondary			Gas	
					Volts	Amperes	Watts		
44	V	80	0.0135	0.00152				*H*	
48*a*	IV	20						Air	
48*b*	IV	40						Air	Watts in Primary,
48*c*	IV	60						Air	
55*a*	IV	50	0.868	0.0309	5200	5.75	100.0	*H*	740
55*b*	IV	50	.594	.0309	5200	—	80.0	*H*	740
55*c*	IV	50	.278	.0309	5200	8.0	—	*H*	
55*d*	IV	50	.155	.0309	5200	9.0	—	*H*	Res. in Secondary, 78 ohms
63*a*	IV	50	.045	.0309	6800	2.35	4.1	*H*	16.9 ohms
63*b*	IV	50	.045	.0309	7000	4.5	9.6	*H*	4.69 "
63*c*	IV	50	.045	.0309	6200	9.5	40.0	*H*	0.5 "
63*d*	IV	50	.045	.0309	6200	16.	115.0	*H*	(leads and spark)
71*a*	IV	50	.0135	.0309	2000 ?	18.	70.	*H*	
71*b*	IV	50	.0135	.0309	4100	18.	100.	*H*	
71*c*	IV	50	.0135	.0309	7000	20.	90.	*H*	
71*d*	IV	50	.0135	.0309	2000 ?	22.	75.	*H*	
75*a*	IV	60	.0135	.0309	5000	18.	80.	*H*	
75*b*	IV	60	.0135	.0309	5000	—	75.	*H*	
75*c*	IV	60	.0135	.0309	5000	—	—	*H*	
75*d*	IV	60	.0135	.0309	5000	—	—	*H*	
79*d*	IV	30	.0135	.0309	5000	15.	—	*H*	
79*e*	IV	60	.0135	.0309	5000	—	—	*H*	
79*f*	IV	25	.0135	.0309	5000	15.	—	*H*	
85*a*	IV	10	.0135	.00543				*H*	Watts in Primary,
85*b*	IV	15	.0135	.00543				*H*	
85*c*	IV	30	.0135	.00543	2700	11.5	85	*H*	520
85*d*	IV	50	.0135	.00543	—	—	—	*H*	
85*e*	IV	70	.0135	.00543	—	—	—	*H*	
91*a*	IV	50	3.0	.0388	—	2.8	27.	*H*	140
91*b*	IV	50	3.0	.0388	4100	6.8	120.	*H*	720
95*a*	IV	40	25.0	.170	—	2.0	—	*H*	Res. in Secondary, 4.9 ohms
95*d*	IV	20	0.0135	.170	—	16.	—	*H*	
99*a*	—	50	25.0	.0388	4100	2.7	60.	*H*	4.9
99*b*	—	50	0.0135	.0388	4100	22.0	125.	*H*	
99*c*	—	50	0.0135	.0388	4100	—	150.	*H*	
99*d*	—	50	25.0	.00543	—	1. ?	45.	*H*	4.9
103*a*	IV	50	25.	.1286	—	1.5 ?	38.	*H*	4.9
103*c*	IV	50	6.46	.1286	—	—	—	*H*	2.59
103*d*	IV	20	0.0135	.1286	—	—	—	*H*	
107*a*	—	100	25.0	.00543	3200	1. ?	78.	*H*	4.9
107*b*	—	100	25.0	.0309	4000	2. ?	58.	*H*	4.9
107*c*	—	100	6.46	.00543	3800	2.	68.	*H*	2.59
107*d*	—	100	0.0135	.00543	4100	14.	95.	*H*	
107*e*	—	100	.0135	.002	4100	10.5	100.	*H*	
112*a*		50	140.0	.002	—	1.4	55.	*H*	9.30
112*b*		50	140.	.00543	—	—	30. ?	*H*	9.30
112*c*	IV	50	140.	.0309				*H*	Res. in Secondary, 9.3
112*d*	IV	50	140.	.1286				*H*	
116*a*	IV	40	140.	.002		1.0	35.	*H*	9.3
116*b*	IV	40	140.	.00543		0.45	35.	*H*	9.3
116*c*	IV	40	140.	.0309		.4	50.	*H*	9.3

PLATE V



3720 3740 3760

No. 127 in *H* at 100 atmos. Ind. 140. Cap. 0.0054
44 in *H* at 80 atmos. Ind. 0.0135. Cap. .0015
190 in CO_2 at 100 atmos. Ind. 0.0135. Cap. .0054

3720 3740 3760

No. 132 in *H* at 30 atmos. 500 Volts. *Fe* Arc.
134 in *H* at 30 atmos. Spark. Ind. 140. Cap. 0.223
137 in *H* at 70 atmos. Spark. Ind. 140. Cap. 0.223

3720 3740 3760

No. 199 Solar Spectrum
186 in CO_2 at 53 atmos. Ind. 0.87. Cap. 0.027
187 in CO_2 at 50 atmos. Ind. 75. Cap. 0.027

TABLE II.—*Continued*

Spectrum No.	Plate No.	Pressure in Atmospheres	Inductance in Millhenrys	Capacity in Mfds.	Secondary			Gas	
					Volts	Amperes	Watts		
116*d*	IV	40	140.	0.1286		—	—	*H*	9·3
120*a*	IV	70	140.	.002		.6	55	*H*	9·3
120*b*	IV	70	140.	.00543		·4	68	*H*	9·3
120*c*	IV	70	140.	.0309		·45	60	*H*	9·3
126	—	100	140.	.00543		·4	55	*H*	9·3
127	V	100	140.	.00543		—	55	*H*	9·3
132	V	30	500 volt arc					*H*	
134	V	30	140.	.2231			37	*H*	9·3
135	—	50	140.	.2231			—	*H*	9·3
137	V	70	140.	.2231			—	*H*	9·3
143*a*	—	1	Iron arc					Air	Watts in Primary, 330.
143*b*	—	10	0.0135	.0309		9	52	Air	
143*c*	—	15	.0135	.0309		15	54	Air	550.
143*d*	—	20	.0135	.0309	3000 ?	15	58	Air	560.
143*e*	—	30	.0135	.0309	3200 ?	20	—	Air	—
143*f*	—	50	.0135	.0309	—	—	—	Air	—
143*g*	—	5	.0135	.0309	—	—	—	Air	—
143*h*	—	1	Iron arc					Air	—
149*c*	—	30	0.0135	.00543	3000	11	50	Air	Watts in Primary, 500.
149*d*	—	30	0.0135	.00543	2700	11.5	85	*H*	520.
149*e*	—	50	0.0135	.00543	3000	8.5	50	Air	320.
149*f*	—	50	0.0135	.00543	—	—	—	*H*	
149*h*	—	70	0.0135	.00543	—	—	—	Air	
149*i*	—	70	0.0135	.00543	—	—	—	*H*	
161*a*	IV	70	75.0	.2231	—	—	—	*H*	
161*b*	IV	70	23.	.0734	—	—	—	*H*	
161*c*	IV	70	8.85	.0734	—	—	—	*H*	
161*d*	IV	70	0.703	.00543	—	—	—	*H*	
161*e*	IV	70	.703	.002	—	—	—	*H*	
167	IV	70	0.0135	.0309	Exposure time of *a*, *b*, *c*, *d*, and *e*, respectively, 2, 6, 26, 66, and 180 seconds.				
170*a*	IV	50	.0135	.0309	—	—	—	CO_2	
170*b*	IV	40	.0135	.0309	—	—	—	CO_2	
170*d*	IV	15	.0135	.0309	—	—	—	CO_2	
170*e*	IV	10	.0135	.0309	—	—	—	CO_2	
174*a*	IV	50	.0135	.00543	—	—	110	CO_2	
174*b*	IV	50	.0135	.00543	—	—	—	*H*	
174*c*	IV	25	.0135	.00543	—	—	—	CO_2	
174*d*	IV	25	.0135	.00543	—	11	—	*H*	
174*e*	IV	15	.0135	.00543	—	—	—	CO_2	
174*f*	IV	15	.0135	.00543	—	—	—	*H*	
174*g*	IV	10	.0135	.00543	—	—	—	CO_2	
174*h*	IV	10	.0135	.00543	—	—	—	*H*	
174*i*	IV	5	.0135	.00543	—	—	—	CO_2	
186	V	53	.867	.0270	—			CO_2	
187	V	50	75.0	.0270	—			CO_2	
190	V	100	0.0135	.00543	—			CO_2	
199	V	Solar spectrum, same region.							

slows the discharge and causes conditions in the spark to approach more those of the arc. But it is just in the arc that numerous but narrow reversals occur (column 2, Table VI). To be sure, in the case of the arc, the briefness of exposure, and the fact that the spectrum is not a composite of numerous spectra as with the spark (p. 245), accounts largely for the more numerous reversals.

Wave-length.—In Plate V a portion of the spectrum toward the region of longer wave-lengths is shown for Nos. 112, 127, and 137. From these, and from the portions of the same spectrum numbers (also Plate V) in the vicinity of λ 3700, it will at once be seen (as many have observed) that the lines of short wave-length, in general, reverse much more readily than those of longer wave-length. As the character of the discharge becomes more disruptive and less like the arc, the lines of shorter wave-length become more prominent, and new "spark lines" appear. This is generally supposed to be accompanied by more intense heat in the spark. The light producing the more readily reversed lines may, then, originate near the center (hottest part) of the spark, and the surrounding vapor causes the reversal, while for the less readily reversed lines, the surrounding vapor is sufficiently hot to give them as bright lines, hence they do not ordinarily reverse (compare Lockyer's "long" and "short" lines of the arc).

Oscillation period.—Not much can be gained by comparing spectra taken with different oscillation periods, for the reason that in changing the period either capacity (C) or inductance (L), or both, must be changed; since $T = 2\pi\sqrt{LC}$. It was thought, however, that with an exceedingly slow period the current would resemble that of the alternating arc, and so might also the spectrum resemble the arc spectrum. Accordingly, a very large capacity and inductance were used, giving a period of 1/900 second, which does not differ greatly from that of ordinary alternating circuits. Under these conditions No. 133, No. 134, No. 135, and No. 137 were taken at 20, 30, 50, and 70 atmospheres pressure, respectively.

It was next attempted to take arc (500 volts with 100 ohms in series) spectra at these same pressures for comparison, but the apparatus was not well adapted to the use of the arc, so that the best spectrum obtained, No. 132 (30 atmospheres), was hardly good

enough to reproduce. However, by comparing negative No. 132 and No. 134, which was taken at the same pressure but with the spark, it was found that the latter gave somewhat wider lines. The reason for this difference is not apparent, since the value of the current seems to be about 4 or 5 amperes in each case. For with the spark, the frequency multiplied by the capacity in farads (see No. 134, Table II), by the charging potential in volts, and by 4,[1] gives 4.9 amperes;[2] while from the resistance and voltage used, the current through the arc must have been about 5 amperes. This current was too intermittent to be measured.

Direct comparison of effects.—In some cases two spectra show that the effect upon the spectrum of a change in one variable is just about compensated for by a simultaneous change in another variable. Thus No. 95*d* and No. 95*a* are very much alike, showing that the widening effect upon the lines, caused by doubling the pressure, is balanced by the narrowing effect of increasing the inductance two thousand fold. With one-fourth as much capacity, but seven-fifths as high pressure, No. 120*c* is similar to No. 112*d*. Also the effect of increasing the inductance from 0.045 to 25 milhenrys appears to be offset by decreasing the resistance from 78 to 4.69 ohms, as shown by No. 63*a* and No. 99*a*. No. 161*e* and No. 161*a* show that increasing both capacity and inductance a hundredfold gives two and a half times as wide lines, hence capacity has the greater effect. Other similar cases might be cited, but the relative importance of these various quantities in affecting the spectrum can perhaps be better discerned from certain particular cases of maximum effects.

Maximum effects.—In studying the effect of capacity four different spectra were taken with the four different values given (see No. 116), first at 40 atmospheres, then at 50 (No. 112), then at 70 (No. 120). The inductance for all twelve spectra was 140 milhenrys. These spectra show that for a certain value of the capacity a given percentage change of the variable (capacity) produces a maximum percentage change in the width of the lines. This value of the

[1] The factor four is introduced because the condenser not only discharges but charges again in the opposite sense in one-half cycle.

[2] Not mean square, but average current during each group of oscillations.

capacity may then be said to be that at which the maximum effect of capacity change occurs (other conditions being as given). With different inductance, etc., this value of the capacity would probably be somewhat different. The percentage change in the variable (e. g., capacity) divided into the resulting percentage change in line-width may be termed a.

In Table III, column 6, the maximum value of a, found in the above way for the different variables is given, and the variable having the largest value for a, namely pressure, may be called the most important variable.

TABLE III

Spectrum No.	Reproductions Plate	Gas Used	Quantity Varied	Relative Width of Reversed Lines	a	No. Reversed Lines	Capacity	Pressure	Inductance
174e	IV	CO_2	Pressure 15 Atmos-	2	2.0	25	0.00543	15	0.0135
174g	IV	CO_2	Pressure 10 phere	1	2.0	8	.00543	10	.0135
143c	—	Air	Pressure 15 Atmos-	3	1.0	35	.0309	15	.0135
143b	—	Air	Pressure 10 phere	2	1.0	27	.0309	10	.0135
79e	IV	H	Pressure 60 Atmos-	2	1.0	61	.0309	60	.0135
79d	IV	H	Pressure 30 phere	1	1.0	61	.0309	30	.0135
112b	IV	H	Capacity .00543	2	0.4	14	.00543	50	.140
112a	IV	H	Capacity .002	1	0.4	2	.002	50	.140
120b	IV	H	Capacity .00543	3	0.2	19	.00543	70	.140
120a	IV	H	Capacity .002	2	0.2	7	.002	70	.140
55b	IV	H	Inductance .58	2	−.11	62	.0309	50	.58
55d	IV	H	Inductance .14	3	−.11	69	.0309	50	.14
103a	IV	H	Inductance 25.	3	−.08		.1286	50	25.
103c	IV	H	Inductance 6.5	4	−.08		.1286	50	6.5
71c	IV	H	Volts 2000?	3	−.13		.0309	50	0.0135
71a	IV	H	Volts 7000	2	−.13		.0309	50	.0135
174d	IV		H	4			.00543	25	.0135
174c	IV		CO_2	3			.00543	25	.0135
149d	—		H	3			.00543	30	.0135
149c	—		Air	2			.00543	30	.0135
91b	IV	H	Watts 120	4	.1		.0388	50	3.0
91a	IV	H	Watts 27	3	.1		.0388	50	3.0
63b	IV	H	Ohms 16.9	4	−.08	61	.0309	50	0.0450
63c	IV	H	Ohms 4.69	5	−.08	68	.0309	50	.0450
75b	IV	H	Sec. expos'd 120	⅓ to ½			.0309	60	.0135
75c	IV	H	Sec. expos'd 20	⅓ to ½			.0309	60	.0135

DISCUSSION OF RESULTS

Having shown the effect of these variables upon the width[1] of the lines, etc., it remains to correlate these various effects by showing

[1] All causes that widen the lines also increase the intensity of the continuous spectrum.

how the energy in the spark depends upon each variable. But the energy in the spark determines the amount, and largely the condition, of the vapor in the spark, which in turn determines the character of the spectrum.

Widening of spectral lines.—I wish first to refer briefly to some hypotheses that have been advanced to account for widening of lines. Among these may be mentioned (1) change of wave-length due to Doppler effect caused by motion of the light-emitting particles[1] (kinetic gas theory); (2) modification of the vibration period due to the presence of neighboring molecules;[2] (3) limitation of the number of regular vibrations by sudden changes of phase[3] resulting from collisions. To these I will add (4) a one-sided widening due to shift caused by change in the average dielectric constant (p. 253).

Of these, (1) is of first importance in rarefied gases, as pointed out by Michelson[4], and of little importance in my work. For the molecular speeds increase with the temperature, which presumably is not vastly higher in the spark under pressure than in the case of incandescent gases at low pressure, which latter give narrow lines. The other hypotheses would account for the great increase in the width of the lines through the crowding together of the molecules as the pressure, or the rate of generation of the metallic vapor, is increased. Especially important in the present work seems (3), by which Michelson has been able to account fairly well for the widths he obtained experimentally.

With the spark the light may be considered as originating mainly in the hot central "core" of the spark, and that in passing through the less heated vapors enveloping it certain wave-lengths are absorbed, giving rise to the absorption lines, just as in the case of the arc.[5] With the direct-current arc, however, we have an invariable source of light whose spectrum is the same from instant to instant, but with the spark I believe this is far from true.[6] The light pro-

[1] Lippich, *Pogg. Ann.*, **137**, 465, 1870; Lord Rayleigh, *loc. cit.;* Michelson, *loc. cit.*, p. 238.

[2] A. Schuster *Astrophysical Journal*, **3**, 292, 1896.

[3] A. A. Michelson, *loc. cit.*

[4] *Loc. cit.* [5] Jewell, *loc. cit.*, p. 239.

[6] Schuster and Hemsalech, *Phil. Trans.*, **193***A*, 189, 1900.

duced during the first oscillations of a group must pass through less cooler vapor than that produced by the later ones after the vapors have had time to accumulate;[1] and should therefore give narrower absorption lines, and also tend more to give bright lines. The spectrum obtained is then a composite of an infinite number of superposed spectra, giving varying degrees of reversal, and for some lines perhaps even superposing the bright line and the reversed line. The latter would account for the fogging of the reversed lines (on the negative) which makes determination of shift so difficult, and also would account in part for the failure of certain lines to reverse which do reverse in the arc. That the first oscillations of a group produce narrower reversals is proven, I believe, by No. 63*a*, for which a large non-inductive resistance was used to damp[2] the oscillations and cut out the later ones. To be sure, the energy of the spark is simultaneously decreased with increase of resistance, which also tends to narrow the lines.

According to the preceding hypotheses, it follows that the width of the metallic absorption lines should increase with the density of the metallic vapors, whether this density is produced by the pressure of the surrounding gas, by rapidity of evolution of vapor, or by both. The thickness of the absorbing layer—or, in other words, the amount of vapor—is important as regards absorption lines, which are the only lines present in many cases. I shall therefore limit myself to showing how increase of pressure, capacity, etc., influences the amount and density of the vapor.

Clearly the amount of vapor about the spark depends not only upon its rate of production, but also upon its rate of dissipation.

[1] H. Crew has recently shown that the alternating arc in different phases produces different spectra.

[2] From the logarithmic decrement, the relative charge on the condenser at the nth oscillation compared to the first is tabulated (Table IV) for various resistances up to 76 ohms. The capacity, inductance and some of the resistances are the same practically as those used for No. 63. It will be seen that with only 10 ohms, less than half (0.44) of the charge is left on the condenser after one complete period, although 76 ohms is necessary to make the discharge aperiodic. The table shows that the very similar spectra, No. 63*a* and No. 63*b*, must have been produced almost entirely by the first oscillation; and No. 63*c*, by the first five or six oscillations; because 16.9 ohms was used with No. 63*b*, and the table shows that after one full period but 0.25 of the charge remains, after four periods but 0.063.

TABLE IV

RELATIVE CHARGE ON THE CONDENSER AT THE *n*TH VIBRATION

(Inductance and capacity the same as used for No. 63. Period $= 7 \times 10^{-6}$.)

Resistance	When $t = \frac{T}{n=2}$	$t = \frac{2T}{n=4}$	$t = \frac{nT}{2}$
76.0 ohms	0.000		
50.0 ohms	.004	0.000016	
16.9 ohms	.25	.063	
10.0 ohms	.44	.194	
5.0 ohms	.67	.45	$n=8$ 0.002
1.0 ohms	.92	.846	$n=16$ 0.5
0.1 ohms	.993	.986	$n=160$ 0.5

The former increases with the energy; the latter increases with the temperature, with the spark-length, and with *decrease* of pressure (increase of free path). Increase of the energy in the spark through increase of the energy of the primary circuit should, then, increase the width of the reversed lines. Increase of capacity causes a greater amount of electricity to surge through the spark at each oscillation, thus developing more energy and more vapor, and consequently producing wider lines. The increase of energy in the former case is probably brought about mainly through causing more groups of oscillations to occur during each cycle (see footnote 2, p. 231), with but little change in the energy of each group. It is also probable that vapor formed by one group of oscillations is pretty well dissipated before the next group (about 1/1000 sec. later) occurs, while with increase of capacity the increase in energy comes with each group and each oscillation. This difference accounts for the fact that capacity change affects the spectrum more than (direct) energy change does (p. 237).

Increase of pressure not only increases the energy, but also decreases the free path. At the same time, the spark-gap must be made much shorter (p. 226); so that the vapors are more confined and hence the light from the "core" must pass through a much thicker stratum, as may be seen by comparing *A* and *B*, Fig. 8. For these reasons, pressure is the most important variable in affecting the spectrum.

Since increase of inductance (L) increases the period ($T = 2\pi\sqrt{LC}$), it will be seen that this decreases the current, as the oscillating charge is the same, and hence decreases the energy; and narrower lines should result, as found. As the spark-gap must be lengthened to increase the potential-drop over the spark, the escape of the vapor is thereby facilitated (see Fig. 8) so that narrower lines result with increase of potential, as shown in No. 71. On the other hand, increase of potential causes an increase in the charge, and hence in the energy, and therefore might be expected to produce wider lines. Under some conditions I think this might occur, but I have not observed it.

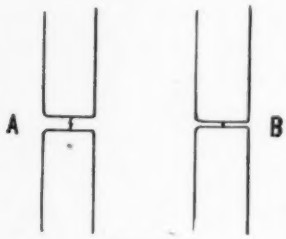


FIG. 8

The effect of the nature of the gas, duration of exposure, and oscillation period have already been discussed on pages 237, 238, and 242, respectively. The actual value of the current during oscillation is too uncertain, and its effect is also too intimately connected with that of energy, to warrant, or require, separate discussion, further than given on pp. 232 and 243.

SHIFTS OF SPECTRAL LINES

Humphreys and Mohler[1] showed that if the gas surrounding the arc is subjected to several atmospheres pressure, the arc lines are slightly displaced relatively to those of the arc at atmospheric pressure.

This displacement, or shift, was determined by them for several lines of nearly all the metals. Since then several investigators[2] have studied the shift of the spectral lines of the spark in compressed gases. Of these Hale and Kent used the highest pressures (1 to 53 atmospheres) and found, for some of the iron lines, shifts as high as 0.22 tenth-meters.

Whether other causes than pressure may produce shift in the case of the spark spectrum is a much-mooted question. Eder and

[1] *Loc. cit.*

[2] J. F. Mohler, *Astrophysical Journal*, 10, 202, 1899; Hale and Kent, *ibid.*, 17, 154, 1903; and others.

Valenta[1] found no such shifts, while Kent[2] and others have found them. Kent has recently repeated his work with great care and is satisfied that such shifts exist. In general, previous investigations have shown that shift increases with increase of capacity and decreases with increase of inductance.[3]

The shifts of about thirty lines have been determined from No. 187, which has fairly well-defined reversed lines, to see if any grouping of lines could be made upon the basis of equal shifts. The shifts of a few lines at pressures of 80 and 100 atmospheres have also been measured.

Measurement of shift.—The shift of the lines of the spark spectrum with reference to those of the arc comparison spectrum taken on the same negative was measured with the dividing engine, upon which was mounted a low-power microscope. In lieu of cross-hairs, a system of fine lines ruled on glass, as shown in Fig. 9, was finally used. The adjustment was so made that some one of these lines was near one edge of the spectral line, while some other one was equally distant from the other edge. By this means more accurate settings could be made than with a single cross-hair, set on the middle. A broken line is far superior to a continuous line, as it permits a nearly unobstructed view of the edge of the spectral line, which is essential with lines that shade off so gradually in intensity.

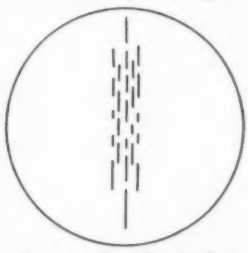


FIG. 9.—Reticle of microscope.

RESULTS

In Table V the shift and width of thirty lines of No. 187 and No. 186 (see Table II) are given, together with shifts obtained by Hale and Kent[4] for some of the same lines at about the same pressure and in the same gas (CO_2, 53 atmospheres). In column 4 *r* and *v* refer to the relative widths of the red and violet components of the bright line on each side of the reversal, *r* (red) being toward the longer wave-lengths. It will be seen that there is but one exception to the rule that the red component has the greater width. This

[1] *Astrophysical Journal*, **19**, 251, 1904.
[2] *Loc. cit.*, p. 221.
[3] Hale and Kent, *loc. cit.*, p. 221.
[4] *Loc. cit.*, p. 221.

is due partly to shift and partly to unsymmetrical widening of the bright line. In all cases the shift refers to the reversed (i. e., absorp-

TABLE V

Wave-Length of Line	Width in Tenth-Meters			Shift									
				For Plate No. 187 Determinations						Hale and Kent	Humphreys and Mohler	For Plate No. 186	
	Emission Line	Absorption Line	r^* and v^*	I	II	III	IV	V	Average			Shift	Width of Absorption Line
*f*3687.62	0.53	0.03	$r=1$ $v=1$	0.089	0.121	0.138	0.116	0.071	0.107			0.123	0.20
*g*3709.37	0.67	.11	$r=3$ $v=2$	.126	.112	.133	.106		.119				
*f*3722.71	0.47	.13	$r=4$ $v=1$	.064	.035								
*e*3733.47		.09		.04	.040		.040						
*c*3758.38	1.33	.212	$r=3$ $v=2$	.094	.076	.093	.082	.102	.089			.127	.60
*d*3763.94	0.93	.13	$r=2$ $v=1$	.084	.064	.085	.085	.093	.082			.135	.35
*h*3765.71	.27	0		.095	.090	.116	.102	.080	.097	0.12		.180	.18
*e*3767.33	.80	.08		.118	.104	.108	.111	.133	.115			.143	.25
3805.49	.27	0		.076	.093		.058	.061	.068				
*h*3813.17	.27	0			.076	.065	.089	.065	.074			.110	.20
*e*3815.99	1.20	.13	$r=2$ $v=1$	.091	.109	.112	.102	.093	.104	.14		.158	.35
*c*3824.63	0.66	.13	$r=2$ $v=1$	.058	.53	.054	.038	.055	.052				
*e*3827.97	1.10	.13	$r=3$ $v=2$	.100	.121	.136	.129		.126	.20		.175	.22
*d*3834.42	1.00	.16	$r=3$ $v=2$	.092	.096	.90	.105	.092	.095			.149	.32
*c*3856.52	0.66	.08	$r=3$ $v=2$	.043	.052	.053	.066	.061	.055			.085	.25
*h*3865.70		0		.076	.064	.081	.093	.060	.075			.160	.23
*g*3969.39	.50	0		.110	.080		.085		.092			.149	.26
*h*3977.91	.22	0				.082	.090	.054	.075				.24
*h*3997.55	.27	0				.081	.085	0.71	.079		0.129		.22
*h*4022.02	.20	0		.077	.090	.085	.097	.118	.093			.180	.64
*h*4030.89	.22	0					.106						
*h*4033.22	.18	0				.074	.077		.075				
*c*4045.98	1.60?	.18	$r=3$ $v=2$			.104	.113	.102	.106		.089	.170	.40
*e*4063.75	1.	.1		.100	.118	.112		.090	.105	.18	.102		
*e*4071.90	0.9	?			.128	.134			.131				
*h*4118.72	.27	0				.106							
*h*4132.20				.103	.112				.107			.195	.22
*h*4156.93	.20	0		.057	.087	.076		1.	.073				
*h*4199.26	.70	0		.085	.094	.092		.088	.090		.120	.180	.20
*h*4219.52	.27	0		.088	.114	.106		.079	.097		.085		

(* When a bright line has superposed upon it an absorption line, r and v, respectively, refer to the relative widths of the red and violet components of the bright line.

tion) line, excepting for those lines of No. 187 whose width of reversal (column 3) is marked zero.

Column 12 gives the calculated shift for the arc at 50 atmospheres, based upon shifts obtained by Humphreys and Mohler[1] at 10 or 12 atmospheres and upon a linear relationship between shift and pressure which they believe to exist. There is fairly good agreement between these calculated values and those determined by me with the spark at 50 atmospheres (column 10). The large capacity and inductance of No. 187 gives a frequency of only 3570, which is not vastly higher than sometimes used with the alternating arc, so that similar shifts might be expected (see p. 242).

It will be seen that the shift obtained by Hale and Kent (column 11) is uniformly larger than that of No. 187, and smaller than that of No. 186. When taking No. 186, a considerably larger capacity was used than that employed by Hale and Kent, which would account for the greater shift (due to unsymmetrical widening of lines); while No. 187, likewise with larger capacity, had also several thousand times as large inductance, which may account for the smaller shift (large inductance produces narrow lines, hence less of false shift due to unsymmetrical widening).

In Table VI the lines marked with a (+) are reversed. It will be seen that in No. 112*a* but two lines, λλ 3720.08 and 3737.28, are reversed; so that these may be termed the most easily reversed and are designated by *a* (column 1). Those reversed in No. 143*h* (the arc) and not in any others are designated by *g*. Those not reversed in the arc may be grouped as class *h*. It was thought that similar lines might be similarly shifted. In column 10, Table V, there are five lines with shifts from 0.073 to 0.075, which all belong to class *h*. To determine whether or not this accords to some law of shift of the lines of a series of Kayser and Runge, will require more experimental evidence than is at present available.

In Table VII are given shifts for some of the lines of plates Nos. 137, 44, 127, and 190, at pressures of 70, 80, 100, and 100 atmospheres, respectively. With No. 190, CO_2 was the gas used. At 100 atmospheres CO_2 is a liquid, in which it was found impossible to run the spark long enough to obtain a good spectrum. By heating

[1] *Loc. cit.*, p. 222.

TABLE VI

Wave-Length	Arc. No. 143h	No. 113c	Arc. No. 143g	No. 170c	No. 120b	No. 112b	No. 120a	No. 112a	Wave-Length	Arc No. 143h	No. 112a	No. 143g	No. 170c	No. 120b	No. 112b	No. 120a	No. 112a
*f*3680.00	+	+							*d*3834.42	+	+	+	+	+			
*f*3687.62	+	+							*e*3841.20	+	+		+				
*d*3705.00	+	+	+		+				*c*3856.52	+	+	+	+	+	+		
*g*3709.37	+		+						*b*3859.50	+	+	+	+	+	+	+	
*a*3720.08	+	+	+	+	+	+	+	+	*h*3865.70								
*f*3722.71	+	+	+						*d*3878.15	+	+			+			
*e*3727.79	+	+		+					*c*3886.43	+	+	+	+	+	+		
*e*3733.47	+	+	+	+					*f*3899.85	+	+	+					
*b*3735.01	+	+	+	+	+	+	+		*h*3922.5		+						
*a*3737.28	+	+	+	+	+	+	+	+	*g*3969.39	+							
*b*3745.50	+	+	+	+	+	+	+		*h*3977.91								
*e*3748.41	+	+	+	+	+	+			*h*3997.55								
*b*3749.63	+	+	+	+	+	+	+		*h*4022.02								
*c*3758.38	+	+	+	+	+	+			*h*4030.89								
*d*3763.94	+	+	+	+	+				*h*4033.22								
*h*3765.71									*c*4045.98	+	+	+	+	+	+		
*e*3767.33	+	+	+	+					*e*4063.75	+	+	+	+				
*h*3813.17									*e*4071.90	+			+				
*e*3815.99	+	+	+	+					*h*4118.72								
*b*3820.59	+	+	+	+	+	+	+		*h*4132.20								
*c*3824.63	+	+	+	+	+	+			*h*4156.93								
*c*3826.02	+	+	+	+	+	+			*h*4199.26								
*e*3827.97	+			+					*h*4219.52								

the compression tube to 38°, which is well above the critical temperature for CO_2, a fairly good spectrum was finally obtained after about two weeks' work. Only a few of the lines on these negatives are well enough defined to permit of shift determinations, but they were the best obtainable at such high pressures.

The shifts on No. 190 are larger than those of No. 127, although both at 100 atmospheres. For No. 127 hydrogen was the gas used,

TABLE VII

Å	Plate No. 190		Plate No. 127		Plate No. 44		Plate No. 137		Plate No. 187	
	Shift	Width	Shift	Width	Shift	Width	Shift	Width	Shift	Width
3763.94...	0.231	0.75			0.131	0.40			0.082	0.13
3767.33...	.264	.80	.44	0.70	.150	.60			.115	.13
3856.52...			.116	.60	.109	.35	.088	.75	.055	.08
3969.39...	.292	.60								
4063.75...	.257	.65					.124	.50	.105	.1

Shift and Width are expressed in tenth-meters

and the shifts are about the same as those of No. 187 at 50 atmospheres in CO_2. Probably as a result of greater shift of the emission line than of the superposed absorption line, the violet component of the emission line is sometimes wanting.[1] The following lines of No. 170*b* are of this class:

3977.89	4107.65
4009.86	4156.97
4014.68	4175.81
4022.02	4219.52

To account for shift, Schuster[2] has suggested that the light-emitting particle may have a period slightly greater than its free period because of the proximity of other particles of like period. Also, according to the electromagnetic theory of light, the atom may be viewed as an infinitesimal Hertz oscillator whose capacity will increase if the dielectric constant ϵ, of the surrounding medium is increased.[3] An increase of ϵ may be brought about by increase of pressure as well as by changing from hydrogen[4] ($\epsilon = 1.000264$) to CO_2 ($\epsilon = 1.000946$); for if

$$\epsilon = 1 + a,$$
$$\epsilon_1 = 1 + Pa,$$
$$= 1 + P(\epsilon - 1),$$

where ϵ and ϵ_1 are the dielectric constants of a gas at one, and at p atmospheres pressures, respectively. Calculating upon this basis and from $T = 2\pi\sqrt{LC}$, the shift of the lines of No. 127 (100 atmospheres pressure) for λ 4000 should be about 72 tenth-meters. The observed shifts are only about 0.15 tenth-meters.

If, however, we postulate a certain region B about the atom into which no other atoms can enter, we thus preclude variation of ϵ in just that region whose ϵ most affects the capacity of the atom. This might account for the above great discrepancy between the observed and the calculated shifts. A metal for which B is large would give correspondingly small shifts. My results point to greater shifts with the gas of greater ϵ, namely CO_2; still it seems that in the dense metallic vapors in the spark, few CO_2 atoms will

[1] Hale and Kent, *loc. cit.*, p. 221.

[2] *Astrophysical Journal*, **3**, 292, 1896. [3] *Ibid.*, **5**, 210, 1897.

[4] L. Boltzmann, *Pogg. Ann.*, **155**, 403, 1875.

mingle, so that the value of ϵ_1 will depend rather upon the proximity of neighboring metallic atoms.

In applying what we know of finite condensers to a supposed one of atomic dimensions, we verge on the purely speculative. This theory, however, may yet be shown to be quite tenable.

SUMMARY

It has been shown that the lines become wider with increase of capacity,[1] pressure,[1] and energy, and narrower with increase of inductance,[1] resistance, voltage, and time of exposure. The probable cause of widening in all cases is an increased amount of vapor about the spark, which depends upon the rate of production of the vapor (which in turn depends upon the watts expended in the spark during oscillations), and upon its rate of dissipation. When CO_2 is used in the compression chamber, wider lines are obtained than with air, but narrower than with hydrogen. Estimates of the probable value of the current, watts, potential-drop, and resistance of the spark during oscillations have been made, based upon the observed (average) values.

The shifts for thirty different lines have been determined and found to be about the same as those measured by Humphreys and Mohler (with the arc), if reduced to the same pressure. Shifts have been determined for pressures up to 100 atmospheres in CO_2 and hydrogen, the latter giving the smaller shifts.

In conclusion I wish to thank Professor Mendenhall for the interest he has taken in the work, and Professor Snow for placing at my disposal all the necessary material and apparatus. I also take great pleasure in acknowledging my indebtedness to Mr. H. W. Kircher, for faithful assistance and valuable suggestions during the early part of the work; and to Mr. R. J. Wallace, photophysicist at Yerkes Observatory, for numerous suggestions in preparing the positives for the photo-engraver.

PHYSICAL LABORATORY, UNIVERSITY OF WISCONSIN,
August 6, 1906.

[1] Hale and Kent, and others, had already observed these effects of capacity, pressure, and inductance, but with smaller values and through much smaller range.

PHOTOGRAPHIC OBSERVATIONS OF GIACOBINI'S COMET (1905 *c*)

By E. E. BARNARD

When near perihelion this comet was unfortunately situated for photographic observation. It was low in the east and was involved in dawn and daylight; when in a very active condition it disappeared in the direction of the Sun. In the latter part of December and the first part of January the comet began to show a strong activity and shot forth a tail 8° or 10° long. To the naked eye it appeared like a hazy star of the fourth or fifth magnitude. If the position had been favorable for observation, the comet would doubtless have become one of the most interesting yet observed. Despite the unfavorable situation, if the morning skies had been clear, a valuable series of photographs would have been obtained. The weather here, however, was unusually unfavorable, and the mornings, with but few exceptions, were overcast. Several photographs were secured, however, on the few mornings the comet could be seen. These pictures are extremely interesting, and I believe they are important. If photographs of the comet were made elsewhere on the same mornings, sufficient distinctive features were visible to give good data for determining the motion of the particles of the tail. Some of these phenomena were much like those of Swift's comet of 1892, while others seemed peculiar to this comet.

On December 7, 1905, the night following the discovery of the comet by Giacobini, it was photographed here with the 10-inch Brashear doublet, but it did not show any features of interest; a faint tail was visible for about ½°. The comet at this time was faint in the 5-inch guiding telescope.

The next photograph was made on December 25. On this plate the tail was about 4° long, and was faint and slender.

On December 29 the tail was remarkably and beautifully developed. The photograph on this date (Plate VI) is in some respects quite unique. From a rather large head and a slender neck the tail widens out on each side in a graceful curve, which partly closes

in again and gives a strong convexity to the tail. The edges of these convexities are sharply defined and are outlined by a rather narrow bright rim or border. The appearance of the tail strongly suggests a hollow convex transparent cone with a sensible thickness. A straight-edge placed along the sides of the tail shows this convexity strikingly, as may be seen on an examination of the plate. I have not noticed quite this appearance before in a comet's tail. Besides this peculiarity of convexity, there is considerable structure in the tail. Several very faint threadlike streams diverge backward from the head on each side at angles of approximately 15°, 20°, and 30° to the axis of the main tail. On the south side, about 1° back from the head, is a long diffused strip running out from the edge of the tail at a very slight angle. This strip seems to cross the bright rim on to the body of the tail, as if it were nearer to us. The entire length of the tail in the picture is about 4½°.

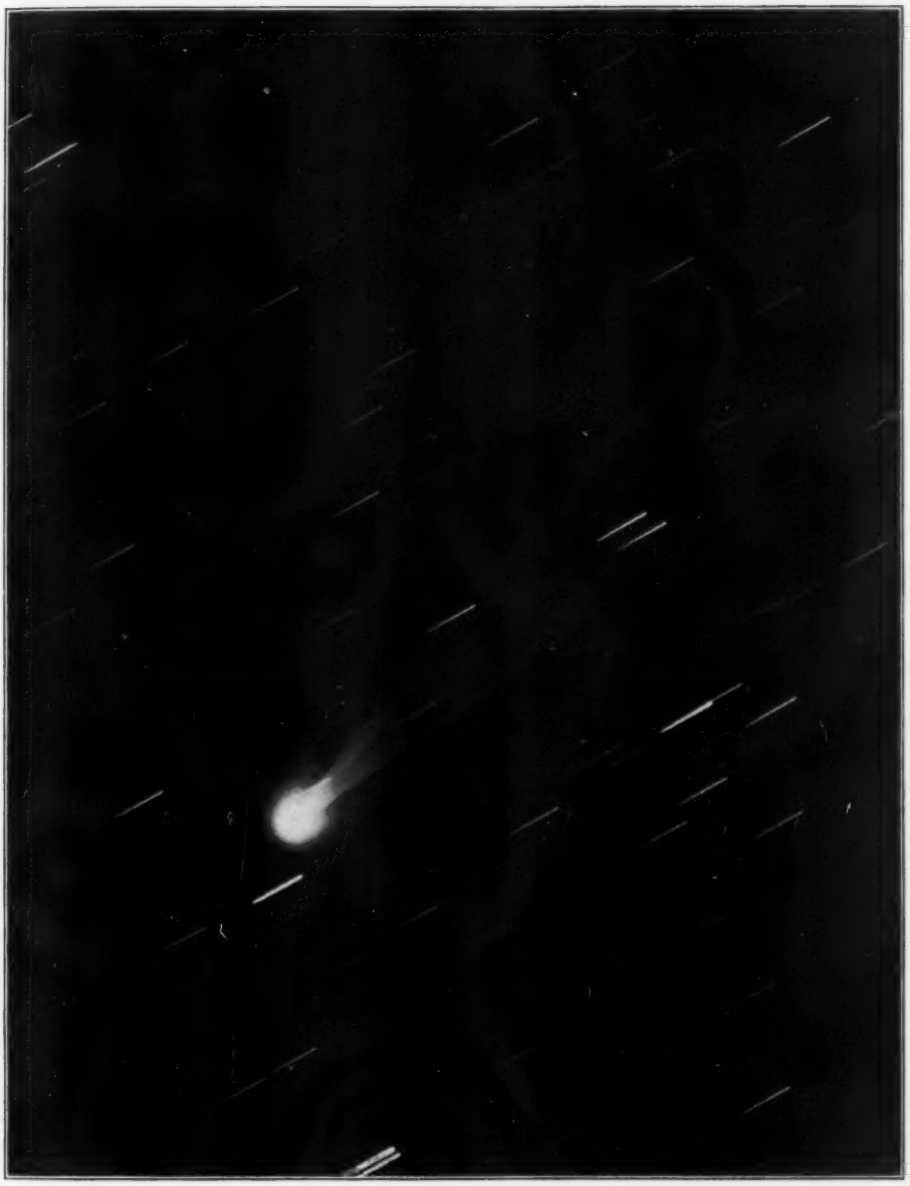
On the night following this, December 30, the head is small, and the first part of the tail seems to be made up of a great number of threadlike strands which diverge from the sides near the head; while the main part of the tail, at first narrow, spreads out in a diffused manner at a large angle from about 1° back from the head. The tail is almost bifurcated in places. The entire comet in this picture differs widely from its appearance on December 29.

No other photograph was possible until January 4, 1906, on which date the plate was much affected by daylight. The tail consists of one main strand, which slightly widens as it recedes from the comet. On the east side there is a slender lateral tail diverging only slightly from the main tail, which is about 8° long.

January 5.—The tail can be traced to the edge of the plate, a distance of 10¼°. In this picture the brightness of the central part of the tail is irregular and wavy. There is at least one slender streamer on the west side of the comet. The exposure was closed in this picture when the Sun's center was 16¼° below the horizon. Even under these conditions the photograph was affected by the dawn.

January 7.—This is a very interesting photograph (Plate VII). The head is very slender; the tail widens out rapidly and seems to be made up of a great number of strands. About 1° from the head is a great deal of beautiful thread-like structure. At this point the

PLATE VI



GIACOBINI'S COMET (1905 *c*)
December 29, 1905, at $23^h\ 20^m$ G. M. T.
Exposure $1^h\ 38^m$. Enlarged 2.5 times. Scale: $1° = 63.5$ mm.

tail is separated into three streams, the central one being very long with irregular masses on it.

January 8.—This plate is badly fogged by daylight. The tail is at least 10° long and irregular; the central brightness alone shows; there is a slender thread on the east side which does not diverge much.

January 9.—This picture was killed by daylight. Very little of the tail shows.

When in these notes reference is made to the length of the tail, it refers to the photograph taken with the 6¼ inch lens.

Following is a list of all the exposures made here with the 10-inch and 6¼-inch lenses of the Bruce telescope:

1905 Dec.	7	
	25	
	29	
	30	
1906 Jan.	4	
	5	
	7	
	8	Injured by daylight
	9	Ruined by daylight
Feb.	21	

The exposure on February 21 was after the comet had passed perihelion, and shows only a suggestion of a tail.

An inspection of these photographs shows that, though the tail was subject to great physical changes, there were no deflections of its general direction. This will be seen by the following table of position angles. These values were obtained by locating the direction of the tail on the *B.D.* charts. I have also taken from these charts the position of the comet's head at the time of each photograph. They refer to the epoch of 1855.0.

TABLE OF POSITIONS OF THE COMET, AND OF THE POSITION ANGLES OF ITS TAIL

Central Standard Time		α	δ	P. A.
1905, December	25.....	$17^h\ 6^m$	$16^h\ 7^m$	$+9°\ 0'$	$317°$
" "	29.....	17 20	16 34	+5 12	312½
" "	30.....	16 58	16 41	+4 24	312
1906, January	4.....	18 2	17 18	−1 11	312
" "	5.....	17 37	17 25	−2 20	312
" "	7.....	17 45	17 42	−4 30	312½

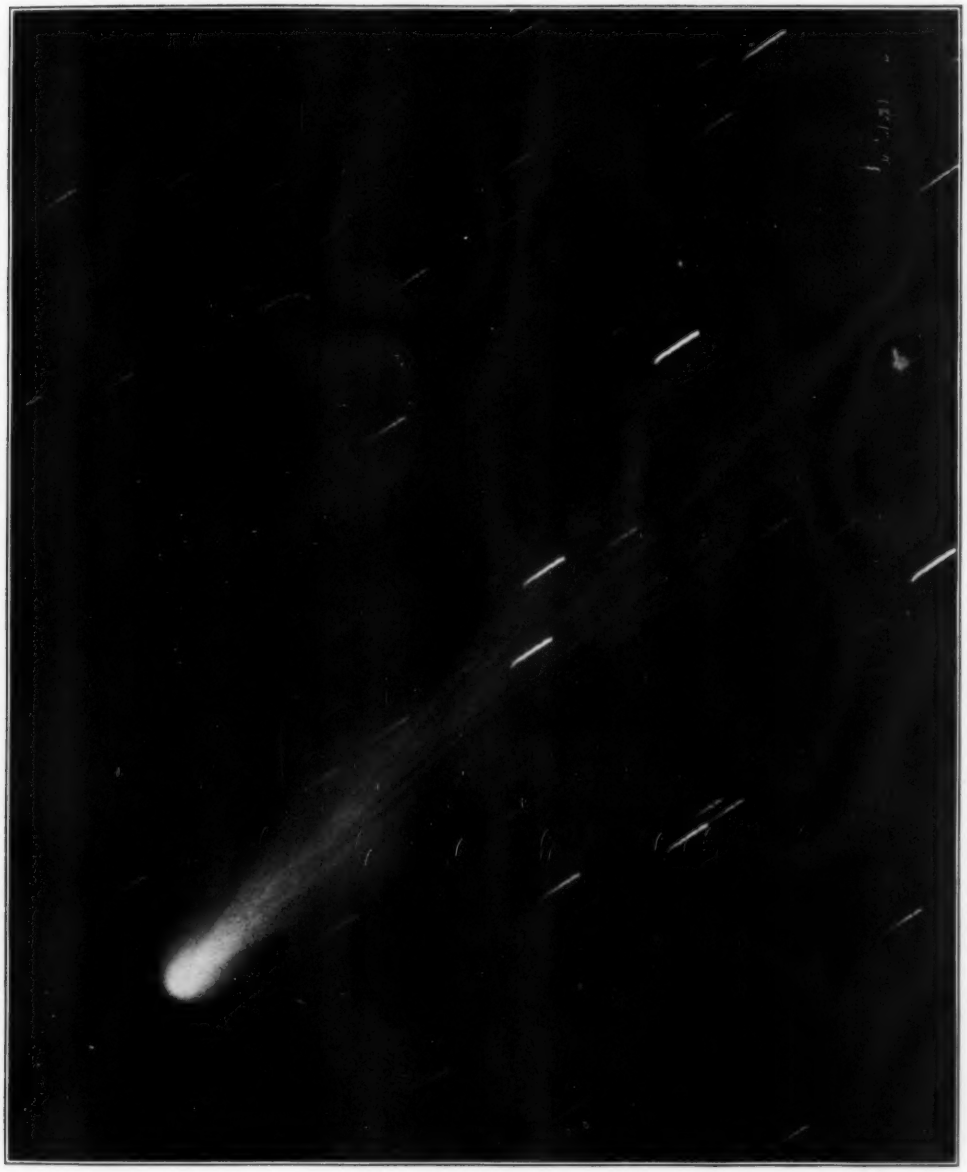
I think there is no question that all the phenomena of this comet were due entirely to the action of the Sun. There does not seem to be any evidence of any outside influence to distort and rupture the tail, as was so evident in the case of Brooks' comet (1893 IV). Indeed, the more I see of comet photographs and of the changes they show in a comet's tail, the more I am convinced that the phenomena of the tail of Brooks' comet were unique, and that they were due to a disturbing influence foreign to the comet and the Sun.[1]

It is a very serious question, in photographing a comet near the Sun, as to when the exposure can begin, if in the evening sky, and when it must cease, if in the morning. One's judgment is rather fickle, for there is always the desire to get as much exposure on the comet as possible. This sometimes leads the observer to carry the exposure too far, and he not only does not get more on his picture, but he may lose that which he has already secured. I have thought that a table might be constructed with the Sun's distance below the horizon as an argument to guide one in closing the exposure in the morning, or beginning it in the evening. But this would probably do more harm than good, for everything would depend on the proximity of the comet to the position of greatest dawn effect, and also on the purity of the sky. Each succeeding morning would bring the comet on a brighter background, if approaching the Sun, at corresponding moments of time, and hence dependence on such a table would be apt to result in ruin of the plate by overexposure of the bright sky. The only guide would therefore seem to be the observer's judgment of the appearance of the sky at the time. This can be fairly relied on, if much work of this kind is done, which we know is not often the case—for the want of opportunity. The best plan to pursue is to so regulate the exposure as to give rather less than the plate will bear, otherwise what has already been obtained is likely to be lost.

YERKES OBSERVATORY,
August 1, 1906

[1] See *Astrophysical Journal*, **22**, 249–255, November 1905.

PLATE VII



GIACOBINI'S COMET (1905 *c*)
January 7, 1906, at $23^h\ 45^m$ G. M. T.
Exposure 50^m. Enlarged 3.4 times. Scale: 1°=80.4 mm.

THE PERIOD OF β *CEPHEI*

By EDWIN B. FROST

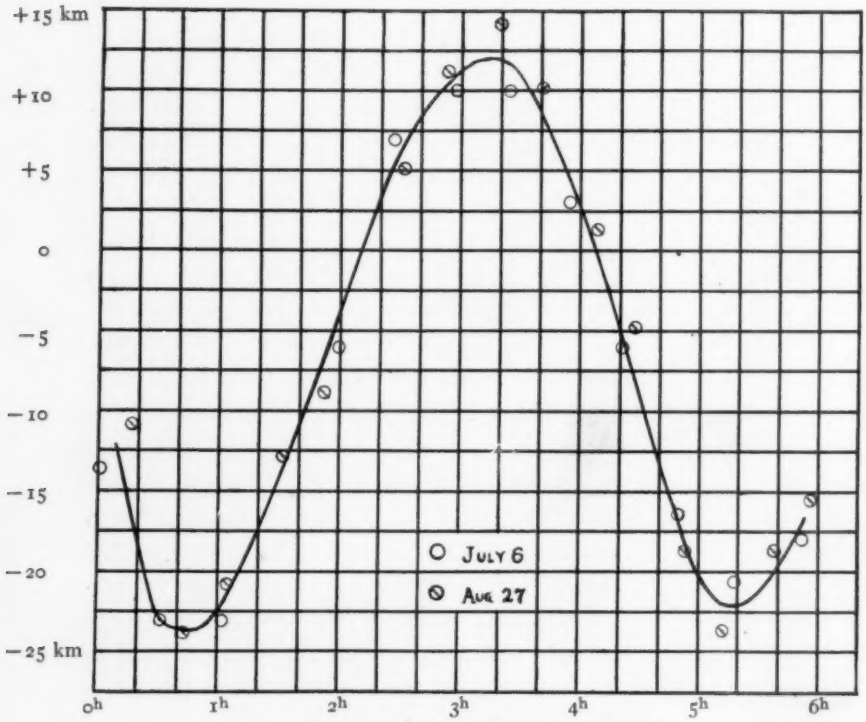
The period of the spectroscopic binary β *Cephei* has been under investigation here, intermittently, for the past four years. The results of the measurement of the first eleven spectrograms, taken between December 18, 1901, and May 23, 1902, were given in this Journal for June 1902 (**15**, 340). Although I inferred from two plates taken on the night of May 14, 1902 that the period was short, the subsequent scattered observations could not be reconciled with periods of 3/4 day, or 3/2 day, or 3 days, which seemed to be indicated by certain plates.

During 1903, 1904, and 1905 the star was seldom observed, a total of only twelve plates being secured. On resuming the observation of the star this season with a view to the settlement of the question, I arranged to have several plates taken on each night, and we were fortunate in obtaining a total of eight plates on the successive nights of May 18, 19, 20, and 21. Approximate measurements of these plates confirmed my suspicion that the period was very short, and indicated that it was very close to 3/16 day or $4^h\ 30^m$.

I then decided to have the star observed continuously through several nights, and the result has been that we (Mr. Barrett or the writer, with the assistance of Mr. Sullivan) secured on May 28, sixteen one-prism plates, with an average exposure of about 12 minutes; on July 6, fourteen two-prism plates, with an average exposure of about 22 minutes; on August 27, twenty two-prism plates, with an average exposure of about 20 minutes. Less complete sets of two-prism plates were obtained on June 18 and June 22.

These plates show conclusively that the star's radial velocity passed through its whole cycle even in a short summer's night. As I have at present no regular assistance in the measurement of spectrograms, a considerable time will elapse before I can measure and discuss our plates, now numbering 110, and I have therefore thought that this preliminary statement might be of interest in view of the exceptional shortness of the period.

In a case like this the exposure-time is a matter of much importance, as it may constitute a considerable fraction of the star's whole period. Of the first forty-six spectrograms obtained here, all but three were taken with a dispersion of three prisms. The exposure varied from 30 minutes, on an especially good night, to 140 minutes on a night when the plate was taken "through thick clouds." A normal exposure for this star under average atmospheric conditions, with three



Provisional Velocity-Curve for β *Cephei*

prisms and the 24-inch (608 mm) camera, would be about 60 minutes. This is nearly one-quarter of the period, so that the early plates cannot give precise epochs for determining the period. For this reason I do not at this time attempt to give the period definitively; but I think it will prove to be very close to $0^d.1904$ or $4^h\ 34^m\ 11^s$.

The effect of the change of velocity during exposure will generally be to diminish the range of velocity and flatten the velocity-curve.

Provisional measures indicate a range of velocity (on the two-prism plates taken with short exposures) of about 34 kilometers, from about +12 km to about −22 km. This implies a velocity of −5 km for the system, and an orbital velocity of 17 km per second.

The accompanying diagram shows a provisional velocity-curve from the observations of July 6 and August 27. The plates of the latter date have been only roughly measured and approximately reduced. The sides of the squares represent 20 minutes of time and 2.5 km of velocity. Inasmuch as the velocity derived from a single plate might, under the circumstances, depart by 5 km from the true value, the agreement of the observations will probably be regarded as satisfactory; although the curve is of course inadequate for a proper determination of the orbit.

Assuming the period to be 274.2 minutes, and the circular orbital velocity to be 17 km per second, the radius of the orbital motion of the bright star would be only about 45,000 kilometers (28,000 miles), as projected upon the line of sight. It is therefore natural to infer that the plane of the orbit is greatly inclined to the line of sight. For instance, if the radius of the orbit of the brighter star is assumed for the moment to be the same as that found by Vogel for *Algol* (1.6 million km), then the inclination of the plane would lack only about 1½° of 90°; and the observed projected velocity would have to be increased nearly forty-fold, yielding an actual velocity of over 600 km per second. Such speculations will be more appropriate after our plates have been fully measured and the orbit has been determined. They tend, however, to imply that the distance of the brighter body from the center of gravity of the system is very small, and they raise the question whether the bodies must not be nearly in contact. Fortunately the period is not a subject of speculation.

On some of the plates, certain lines have the appearance of complexity, and indicate the presence of a second component spectrum. This suggests that the second star may be not more than one or two magnitudes fainter than the brighter star.

The spectrum is an excellent example of the *Orion* type, with fairly sharp silicon lines in addition to those of helium, hydrogen, oxygen, and magnesium, and it may be regarded as well measurable,

in so far as not complicated by the superposition of lines from the second star.

It is certainly astonishing in working on this star to find that plates taken in immediate succession, with the epoch of mid-exposure separated by less than half an hour, show marked differences of radial velocity, at times reaching 10 kilometers.

So far as known to the writer, the shortest period hitherto determined for a spectroscopic binary is 1.45 days for μ *Scorpii*, as found by Professor S. I. Bailey;[1] and for *V Puppis*, as found by Professor E. C. Pickering[2] (the latter star an *Algol*-type variable[3]). We may compare with the period of β *Cephei* that of the variable *W Ursae Majoris*, $4^h\ 0^m$; and that of 14. 1904 *Cygni*, $3^h\ 14^m$, which is the shortest period known for a variable star.

YERKES OBSERVATORY,
October 1906.

[1] *Astrophysical Journal*, **4**, 253, 1896.

[2] *Ibid.*, **4**, 373, 1896; **7**, 139, 1898.

[3] The period of the light-variation of *R Canis Majoris*, found last year to be a spectroscopic binary, is $27^h\ 16^m$; but we have not yet obtained spectrograms enough to establish its period spectroscopically.

ON THE THEORY OF CEMENTED DOUBLETS

By HENRY C. LOMB

The theory of the aberrations of optical systems as developed by the lamented Abbe leaves little to be desired in point of perspicuity and elegance. The simplification accomplished by Abbe is due in great measure to the introduction of a certain function of the geometrical and physical constants of a refraction at a given surface, which function, known as Abbe's Invariant, may be defined as follows.

In Fig. 1, let ρ be the reciprocal of the radius OC of a spherical surface separating two media of refractive indices n and n', and let OA be the "optical axis" about which the portion of the surface considered is symmetrical; further, let σ and σ', respectively, be the reciprocals of the distances OS and OS' at which the incident ray PS (produced) and the refracted ray PS' meet the axis, all these distances being measured from the vertex O of the surface as origin, the direction of the incident light regarded as positive. (In Fig. 1, for example, ρ, σ and σ' are all positive.) Then, for rays near the axis, Abbe's Invariant Q is defined by the relation

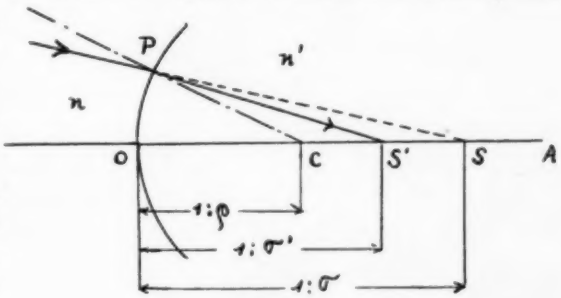


FIG. 1

$$n(\rho-\sigma)=Q=n'(\rho-\sigma') .$$

The name "invariant" refers, of course, to the property of the function remaining invariable or of the same form before and after the refraction. Its physical significance becomes apparent if we remark that Q is another form of the law of refraction

$$n \sin i = n' \sin i' ,$$

where i and i' are small angles of incidence and refraction.

In the following, we shall apply the formulae of Abbe to the case of a thin cemented doublet, i. e., of two thin lenses in contact, the two adjoining surfaces of the lenses having the same curvature. Considering, for the time being, monochromatic light only, the qualities which such an objective ought primarily to possess are:

1. It must have a prescribed *focal length.*

2. It must be free from *spherical aberration;* that is, all rays proceeding from a point S *on* the axis (Fig. 2) must be accurately refracted to a single point S' on the axis.

3. It must fulfil the so-called *sine condition;* that is, rays proceeding from a point P (Fig. 2) at a small distance from the axis and on a perpendicular SP to it, must be brought to a focus at a point P' also on a perpendicular $S'P'$ to the axis, the points S and S' being the same points considered under 2.

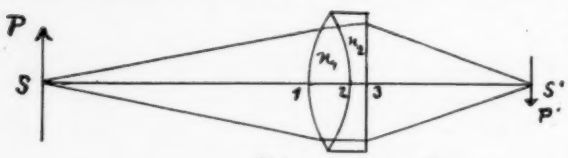


FIG. 2

The fulfilment of 2 is a prerequisite for the fulfilment of 3, and only when both conditions are simultaneously met can a correct image of an object of even moderate dimensions be formed by the lens-system.

We proceed to formulate these conditions analytically. Let ϕ be the reciprocal of the equivalent focal length, that is, the power, of the complete system, ϕ_1, ϕ_2 the powers respectively of its two component lenses. Then

$$\phi=\phi_1+\phi_2 . \tag{1}$$

Employing the notation of Fig. 1, we have, for the spherical aberration of a system of k surfaces,[1]

$$S_{\mathrm{I}}=\sum_{k=1}^{k} Q_k^2\Delta_k\left(\frac{\sigma}{n}\right)=0 \tag{2}$$

and for the sine condition

$$S_{\mathrm{II}}=\sum_{k=1}^{k} Q_k\Delta_k\left(\frac{\sigma}{n}\right)=0 . \tag{3}$$

[1] S. Czapski, *Grundzüge der Theorie der optischen Instrumente nach Abbe*, pp. 115, 148. Leipzig, 1904.

Here Δ_k denotes the variation of the quantity following it, before and after refraction at the kth surface; thus

$$\Delta\left(\frac{\sigma}{n}\right)=\frac{\sigma'}{n'}-\frac{\sigma}{n},$$

the primes indicating the values after refraction.

The sums (2) and (3) are particularly valuable in studying an already existing or an arbitrarily assumed optical system. They are readily computed in connection with the trace of an axial ray, and the influence of each surface on the final image is given by the size and sign of the term which it contributes to the total sum. But in order to discover all possible solutions which simultaneously fulfil some or all of the conditions noted above, it is essential that the expressions which determine them contain not, as here, several unknown quantities, but *one* unknown quantity only, with reference to which the equation can then be solved. For this unknown variable we take the optical invariant of one of the surfaces, namely that of the surface which is common to the two lenses. The particular advantages accruing to this selection will presently appear.

Consider the case of a single thin lens in air, of power ϕ and of glass having a refractive index n. Applying (2) to a system of two surfaces, we have for its spherical aberration,

$$S=Q_1^2\Delta_1\left(\frac{\sigma}{n}\right)+Q_2^2\Delta_2\left(\frac{\sigma}{n}\right). \tag{4}$$

Now, at the first surface of the lens

$$Q_1=\rho_1-\sigma_1, \qquad \frac{Q_1}{n}=\rho_1-\sigma_1',$$

whence

$$Q_1\left(\frac{1}{n}-1\right)=\sigma_1-\sigma_1'.$$

That is,[1]

$$\Delta_1\left(\frac{\sigma}{n}\right)=\frac{\sigma_1'}{n}-\frac{\sigma_1}{1}=\left(\frac{1}{n}-1\right)\left(\sigma_1-\frac{Q_1}{n}\right). \tag{5}$$

Similarly, at the second surface of the lens

$$\Delta_2\left(\frac{\sigma}{n}\right)=\left(\frac{1}{n}-1\right)\left(\frac{Q_2}{n}-\sigma_2'\right). \tag{6}$$

[1] Cf. *l. c.*, p. 117.

Substituting (5) and (6) in (4),

$$S=\left(\frac{1}{n}-1\right)\left\{\sigma_1 Q_1^2-\sigma_2' Q_2^2-\frac{1}{n}(Q_1^3-Q_2^3)\right\}. \tag{7}$$

Further,

$$\sigma_2'-\sigma_1=\phi\,, \quad \text{and} \quad Q_1-Q_2=\frac{n\phi}{n-1}\,.$$

Accordingly, as we eliminate Q_2 and σ_1, or Q_1 and σ_2' from (7), we find, after several reductions, for the spherical aberration of a single lens of power ϕ, the two equivalent forms

$$S=\phi\left\{\left(1+\frac{2}{n}\right)Q_1^2-\left(2\sigma_2'+\frac{3\phi}{n-1}\right)Q_1+\frac{n\phi}{n-1}\left(\sigma_2'+\frac{\phi}{n-1}\right)\right\}, \tag{8}$$

$$S=\phi\left\{\left(1+\frac{2}{n}\right)Q_2^2-\left(2\sigma_1-\frac{3\phi}{n-1}\right)Q_2-\frac{n\phi}{n-1}\left(\sigma_1-\frac{\phi}{n-1}\right)\right\}. \tag{9}$$[1]

Now apply (9) to the first lens of the doublet (Fig. 2) and (8) to the second lens. Then, since the adjacent surfaces have the same curvature, the variable Q will be identical for both lenses, and employing the notation of the figure, we finally derive for the total *spherical aberration of the cemented doublet*,

$$S_{\mathrm{I}}=\left\{\phi+2\left(\frac{\phi_1}{n_1}+\frac{\phi_2}{n_2}\right)\right\}Q_2^2+\left\{3\left(\frac{\phi_1^2}{n_1-1}-\frac{\phi_2^2}{n_2-1}\right)-2(\phi_1\sigma_1+\phi_2\sigma_3')\right\}Q_2$$
$$+\frac{n_1\phi_1^2}{n_1-1}\left(\frac{\phi_1}{n_1-1}-\sigma_1\right)+\frac{n_2\phi_2^2}{n_2-1}\left(\frac{\phi_2}{n_2-1}+\sigma_3'\right). \tag{10}$$

In much the same manner we find for the *sine condition of the doublet*,

$$S_{\mathrm{II}}=\left\{\phi+\left(\frac{\phi_1}{n_1}+\frac{\phi_2}{n_2}\right)\right\}Q_2+\left(\frac{\phi_1^2}{n_1-1}-\frac{\phi_2^2}{n_2-1}\right)-(\phi_1\sigma_1+\phi_2\sigma_3'). \tag{11}$$

Here

$$\sigma_3'-\sigma_1=\phi=\phi_1+\phi_2\,. \tag{12}$$

Equations (10) and (11) are the relations sought; their vanishing carries with it the annihilation of the spherical aberrations in and without the axis, of a cemented doublet having a power of ϕ and having $1:\sigma_1$ and $1:\sigma_3'$ for conjugate focal distances. In the case of the telescope objective for which $\sigma_1=0$ and $\sigma_3'=\phi$, the equations reduce to a still simpler form.

We note that for any one particular value of the parameter ϕ_1 (or ϕ_2) there exist but two doublets which are spherically corrected

[1] (9) may also be obtained from (8) by reversing the direction of the light.

in the axis and but one form which fulfils the sine condition, but the root of (11) need not necessarily be a root of (10). In practice, we are restricted by the limited number of glasses available, so that it is not always possible to completely correct both spherical errors and, simultaneously, other errors (for example those of color) also. In such cases these unavoidable deviations from perfection must be apportioned to the best advantage.

The value of the equations (10) and (11), aside from the favorable form of the coefficients, lies in the fact that each of the spherical aberrations is expressed directly in terms of the powers of the component lenses and the prescribed conjugate focal distances. The symmetry and simplicity of the coefficients are evidently due to the peculiar choice of the unknown quantity, namely the invariant Q_2 of the cemented surface, about which surface the doublet is, in a measure, symmetrical.

The above methods are applicable to other aberrations for which algebraic expressions are known, and they may also be applied to more complicated optical systems by dividing the latter into doublets. But a more detailed discussion of this subject would exceed the limits of this paper.

ROCHESTER, N. Y.,
August 6, 1906.

COLOR-FILTERS FOR ASTRONOMICAL PHOTOGRAPHY WITH REFLECTING TELESCOPES

By ROBERT JAMES WALLACE

The great advantage now taken of photography in recording astronomical data, and the ease with which a visual refractor may be converted to a photographic by means of a color-filter, have gradually changed and enlarged the methods in telescopic work until there is now practically no branch which can not or could not be better performed by its aid.

The lens of the refractor is corrected for a certain limited spectral region—generally, the yellow-green near λ 5550, because in this region the eye is most sensitive to slight differences, the remaining hues coming to an approximate focus at varying distances from this point of correction. The function of the color-filter consists simply in absorbing from the incident light all other hues but those for which the lens is corrected.

Strictly speaking, it is not possible to construct a filter which will accomplish this end by itself without very greatly lowering the luminosity of the hue transmitted, nor is it necessary to do so. Because of the selective sensitiveness of the photographic plate we are able to divide the work between the filter and plate. If, for example, we select such a plate as Cramer's Instantaneous Isochromatic, we find a secondary maximum of photographic action which corresponds to the yellow-green of the spectrum; the insensitiveness of the plate (at normal exposure) to red or orange renders it unnecessary to absorb either of these hues by the filter, but only to absorb the ultra-violet, violet, blue, and to dim down the blue-green, the plate itself being but slightly sensitive to this hue.

In the case of the reflecting telescope the necessity for employing a color-filter has not been recognized to the same extent, because of the fact that all the component rays in white light are brought to a focus at the same plane, and it requires but a limited experience to obtain with this instrument a photographic record of telescopic objects which show structural detail far beyond the ability of the eye to perceive.

There is, however, one very serious defect in the photographs thus obtained; that is, an almost exact reversal of color-luminosity. If we compare the luminosity-curve of the spectrum with the intensity-curve of the ordinary photographic plate, we see in a moment how utterly false and unreliable is the result in so far as color is concerned. Considered even as a representation of *form*, it does not conform to the requirements of a "record of fact," because it is quite within the bounds of possibility to conceive of structural detail visible with a hue to which the plate is altogether insensitive.

This discrepancy between the visual luminosity and photographic intensity is very evident in the case of direct stellar photography, which becomes considerably worse when "extra-focal" images are made use of, as in the later methods of photographic photometry. The use of "isochromatic" plates tends but slightly to a betterment, for, when we consider that such a plate still retains its maximum sensitiveness to the violet end of the spectrum, then it can be seen that conditions are not greatly improved.

In consideration of the needs of this branch of the work, the writer undertook the preparation of a color-filter for use with the 24-inch reflector, to be used initially in obtaining negatives for photographic stellar photometry. Briefly stated, the method employed in adjusting such a filter consisted in (1) isolating a few dyes and making of them a special spectroscopic examination, singly and in combination; (2) coating trial filters with carefully measured amounts of gelatin containing known amounts of dye and estimating the approximate density of the spectrum as photographed through this filter against the normal luminosity-curve; (3) coating the optical glass with the amount of dyed gelatin as thus determined and measuring the density of the spectrum negatives taken through this screen; (4) photographic determination of the absorption at various exposures, and of the exposure increase.

From knowledge gained by a somewhat extended experience in making a large number of filters for various purposes, the following dye-stuffs were selected, estimated as being fairly near to the absorption required, viz.:

Tartrazine
Auramine O.
Metanil yellow S.
Nitrosodimethylanilin

From these dyes color-wedges were now prepared. A solution was made of

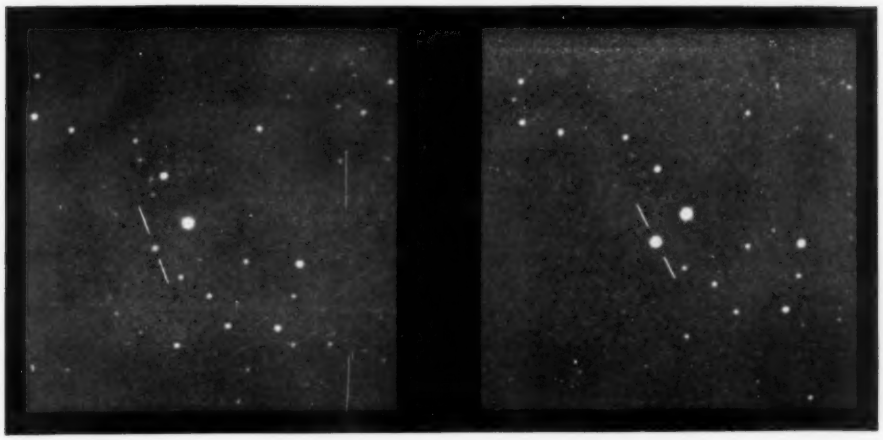
Gelatin (Fischer and Schmidt—extra hard) . .	2.5 grams
Distilled water	100.0 cc
Dye	0.25 gram

Seven cc of this solution at a temperature of 55° C. was flowed upon a plane glass strip 50×250 mm, and then laid aside to set in a drying-chamber, with one end raised to a height of 5 mm from the horizontal plane of the support, thus causing the gelatin to flow slowly toward the lower end. When dry a plane cover-glass was cemented on with Canada balsam and the edges bound. A cm scale was then ruled upon the glass with a writing diamond and a series of spectrograms made showing the absorption at every cm for constant exposure.

At this point it may be well to note the entire unsuitability of the prismatic spectrum for work of this nature. What we are concerned with principally is the rendition of the relative spectral *luminosity*—the dispersion is of no moment, provided it be sufficient to allow of the spectrum being easily read. The difficulty comes in the *interpretation* of prismatic results. We have of course the various dispersion formulae by Helmholtz, Ketteler, Cauchy, Hartmann, and others, but for work of this nature they are of no value whatever, because they do not take into account the absorptive effect due to density and composition of the glass composing the prism itself as it influences the luminosity. For example, supposing that a particular prism is of such a density and absorption that with normal exposure it gives a negative in which the ultra-violet is only impressed to λ 3900, then there is no formula which can supply the photographic intensities of the shorter wave-lengths to which the plate is normally sensitive, while the absorption even throughout the visible portion is still an unknown quantity and varies with every change in the refractive index. As an illustration of this want of reliability, Fig. 1 of Plate VIII shows the influence of this absorption in the comparison between the prismatic and diffraction spectrum in which the lack of concordance may be readily seen.[1]

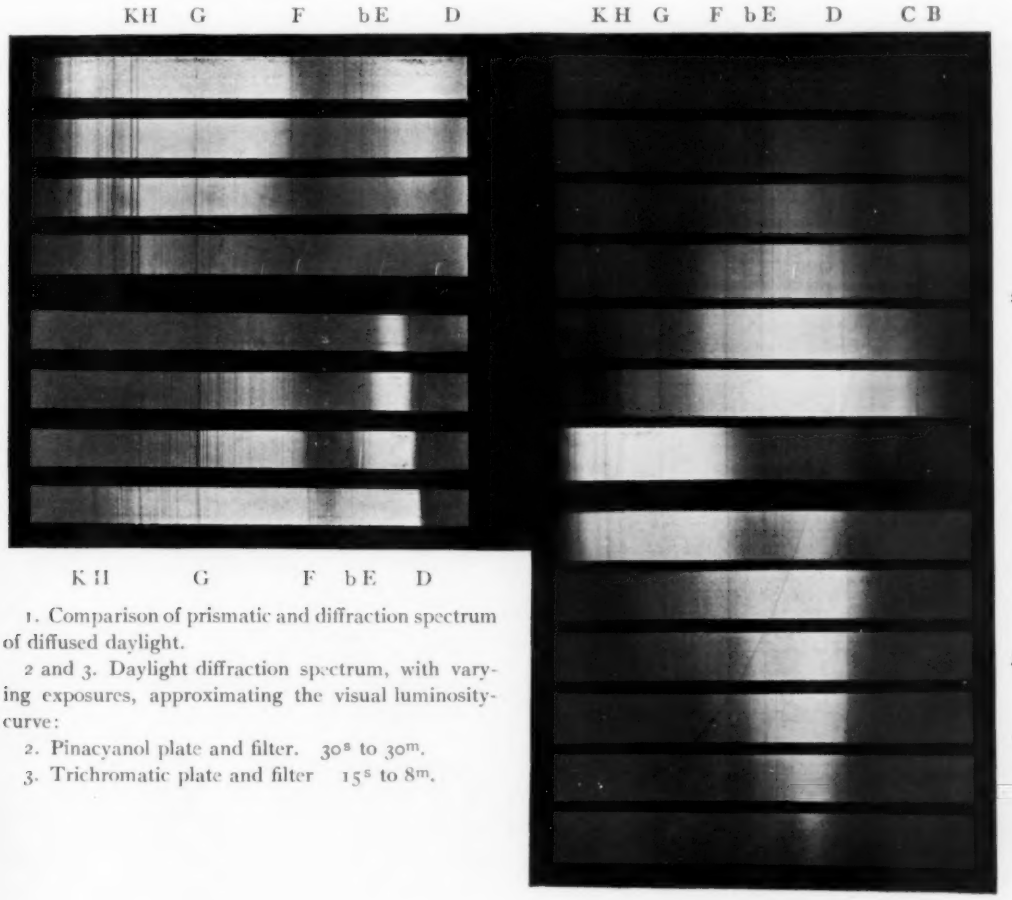
[1] For the prism used $\mu_D = 1.6994$.

PLATE VIII



U CYGNI

4 *a*. On Seed "27" plate, without filter. 4 *b*. On Cramer "Trichromatic," with filter.



1. Comparison of prismatic and diffraction spectrum of diffused daylight.

2 and 3. Daylight diffraction spectrum, with varying exposures, approximating the visual luminosity-curve:

2. Pinacyanol plate and filter. 30^{s} to 30^{m}.

3. Trichromatic plate and filter 15^{s} to 8^{m}.

In all of the spectrographic work, therefore, use has been made of a replica grating of 15,150 lines to the inch, with the collimator directed at a constant angle to the northern sky, and illuminated by diffused light.

The function of the color-filter is to reduce the preponderance of action in the blue and violet region of the spectrum and absorb entirely the ultra-violet. The transmission throughout the remainder of the spectrum should be undimmed by any absorptive action due to the dye.

Examination of the negatives from the wedges, as above outlined, shows that in the dye Tartrazine (Badische Anilin- and Soda-Fabrik) we may obtain the first component of the filter sought for. Fig. 1 of Plate IX shows the record of this color-wedge, in which the ultra-violet transmission will be noted as extending down even into the dense end of the wedge. This transmission, which is masked by the absorption in prismatic spectra, becomes painfully apparent when the exposure is increased or the light rendered more intense, as is shown in Fig. 1*a*. Such a record serves the very useful purpose of showing the danger in the use of this much-vaunted dye for trichromatic and orthochromatic color-filters, even when used in extremely concentrated form. It will be observed that in scale-number 17 of this record the excessive density in the blue of the spectrum from F to G is well corrected for, but that the violet is still too strong, and the ultra-violet is of course transmitted. Another color-wedge, made of

Gelatin	2.5 grams
Water	100.0 cc.
Aesculin	0.2 gram

shows that at scale-number 23 or 24 we have an absorption corresponding to the requirements necessary for the second component of the filter, viz., absorption of the ultra-violet with a gradual absorption in the visible violet. (Plate IX, Fig. 2.)

The two color-wedges are now superposed upon one another with the selected scale-numbers in agreement and an exposure then made through the combination, which gave a result closely approximating the effect sought for. All preliminary exposures and

records are made on Cramer "Instantaneous isochromatic" plates, which have a comparatively low sensitiveness to the blue-green about λ 5050, for which due allowance must be made in the interpretation of the spectroscopic records. It will be evident that any absorption of the dye in the blue-green region would be instantly detected in the photographed spectrum.

Arrived now at a satisfactory point in the trial exposures, the next consideration is the production of a filter which will possess the same absorptive action as do the combined color-wedges at the points selected. Monpillard[1] has suggested a method for obtaining this result, which consists in measuring the thickness of the color-wedge at the selected point (e) and using this as one term of a simple proportion. Two other terms consisting of thickness (e_1) and weight of dye-stuff (d) are obtained by coating a separate plane glass of known area with a measured amount of dyed gelatin; the amount of dye necessary for the finished filter (x) is then to be found from the simple calculation,

$$\frac{e}{e_1}=\frac{d}{x}, \quad x=\frac{e_1 d}{e}.$$

This method commends itself by its simplicity, but in the hands of the writer it has not proven suitable. Presumably, if the thickness of the film on the color-wedge at the selected point were measured more delicately than in the method adopted, say, in the interferometer, then a much closer approximation might be arrived at than has been possible when using a Brown & Sharp micrometer caliper. As it was, the *critical* adjustment of a filter is so delicate that a minute quantity of dye, either in deficiency or excess is fatal to correct performance. The making-up of a large quantity would also tend to reduce error, but for the manufacture of a single filter of special absorption such a course is not practical.

By a method of trial and error we may, however, very speedily arrive at an extremely satisfactory duplication. A plate of ordinary glass of exactly the same size as the desired filter is taken, and upon this is flowed a measured amount of the same solution as was used in making the color-wedge. This amount is approximately deter-

[1] *Comptes Rendus*, **141**, 31–33, 1905.

PLATE IX

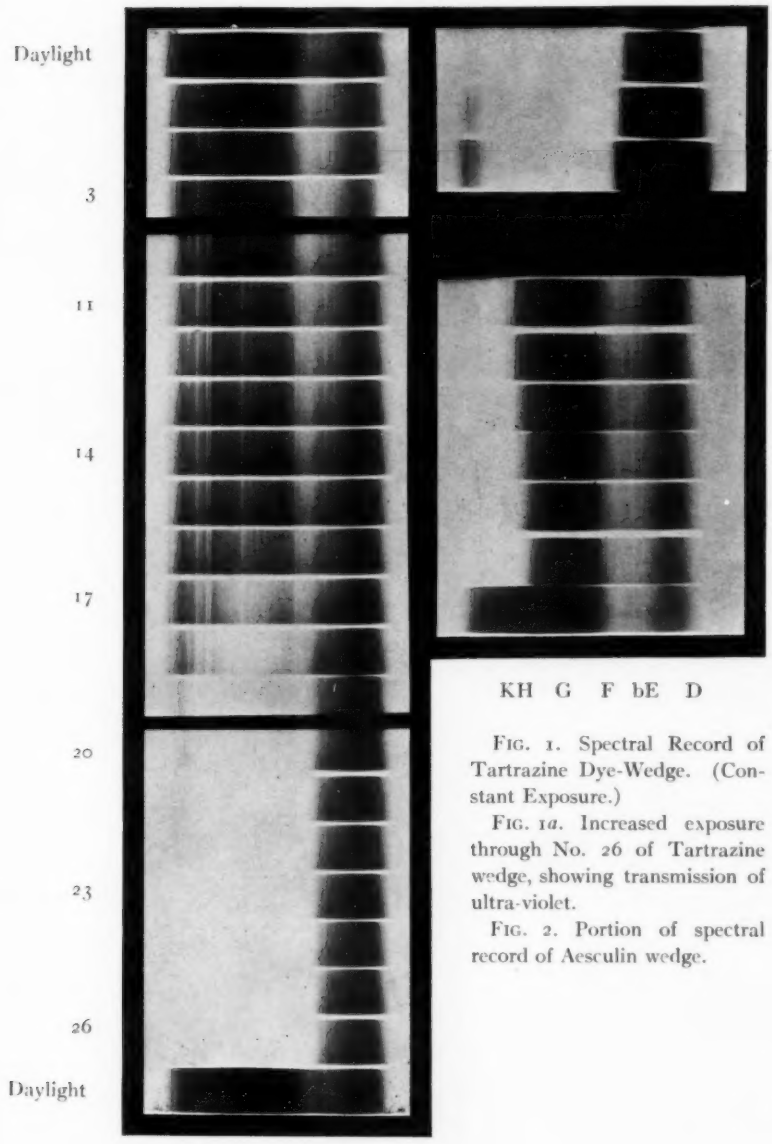


Fig. 1

Fig. 1. Spectral Record of Tartrazine Dye-Wedge. (Constant Exposure.)

Fig. 1a. Increased exposure through No. 26 of Tartrazine wedge, showing transmission of ultra-violet.

Fig. 2. Portion of spectral record of Aesculin wedge.

mined by visual observation, while in contact with a white surface on which also rests the wedge.[1]

As all dyed filters generally dry with a slight shift in absorption toward the red, allowance must be made for this; therefore two other glasses are coated with slightly smaller amounts of solution, and then dried rapidly by fan. Exposures to the spectrum are then made through each and any further correction noted.

In coating the optical glass for the finished filter the drying cabinet (which should be large and roomy) is carefully dusted with a damp cloth, and the supporting plate carefully leveled. The glass being coated with the determined amount, and laid upon the leveled plate, the door is closed, and left so until dry.

In filters of exact adjustment, where there are two components, it will be found advisable to flow each plate separately, as a much closer result can be thus arrived at than by combining the dyes and flowing once.

The solution actually used for coating the glass plates was made up as follows:

Gelatin, 5.0 grams Water, 200.0 cc	} Stock.	
Dye solution A:	Stock gelatin solution,	100.0 cc
	Tartrazine, 0.15 gram	
Dye solution B:	Stock gelatin solution,	100.0 cc
	Asculin, 0.2 gram.	

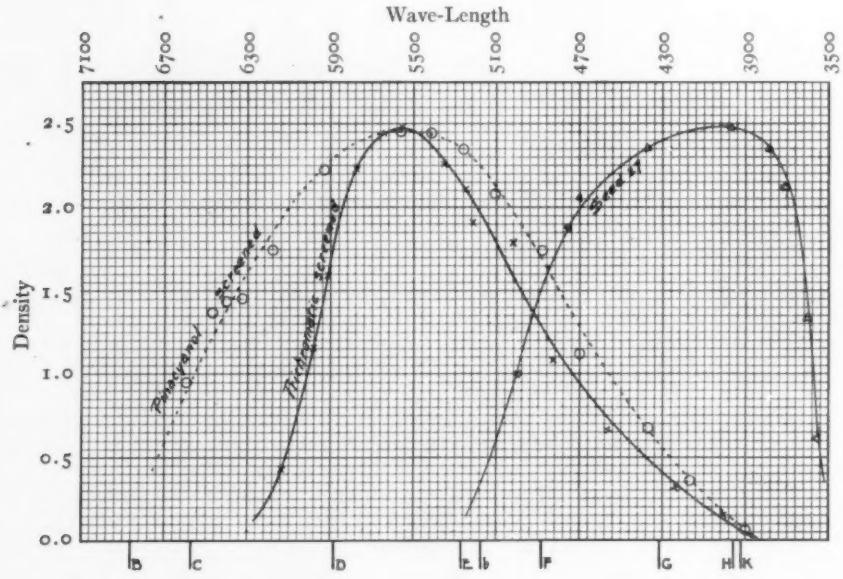
Of solution A 2.5 cc was flowed upon the glass plate of 58 sq. cm area, while 3.6 cc of solution B was flowed upon the cover-plate of similar size. The actual amounts of dye on each surface would then be A=0.00375 gram, B=0.0072 gram.

The process seems lengthy in the repetition, but is in reality quite rapid in performance. Especially is this the case where, as in the laboratory of the writer, the spectrograph, visual spectroscopes (prismatic and diffraction), and spectrophotometer are permanently set up in position for immediate use. The collection of dyes in color-wedge form, with their accompanying photographic records

[1] This method has been adopted after various trials with instrumental methods (colorimeter, tintometer, etc.) for, taking into account the shade change in drying, no greater reliability could be obtained.

ready for consultation, is obviously of inestimable value in many ways.[1]

In the testing of such a filter the first consideration is that of its influence upon the correct representation of the spectrum luminosity. A series of exposures was therefore made upon two plates, a Cramer "Trichromatic," and a Seed "27" bathed in pinacyanol. These negatives were then measured in the spectrophotometer and the curves



plotted; in each case that spectrum selected for measurement gave as its maximum a density[2] of 2.5 (Hurter and Driffield).

The best result, as will be seen from the curves, is obtained by the use of the pinacyanol-bathed plate, which was prepared in a bath of $\frac{1}{300,000}$ for a period of 2½ minutes, and followed by a washing of about 3 minutes; the plates bathed were Seed "27 Gilt Edge." The spectra obtained upon this plate leave little to be desired (Fig. 2 of Plate VIII), but it would be obviously advantageous if use could

[1] All carefully measured filters made by the writer, such as those for the 40-inch Yerkes refractor, the Lowell Observatory, etc., have been derived in a similar manner.

[2] The development of all spectrum exposures is kept constant in constitution of developer, and time and temperature of development.

be made of a commercial plate already prepared and easily obtained. The "Trichromatic" was found to give very favorable results (Fig. 3, Plate VIII), where the extreme red was not required, although the lowering of sensitiveness in the blue-green at λ 5100 is still apparent, as will be seen from the measurement points on the mean curves.

In the series of varying time exposures upon this plate, it will be noted that the point of maximum intensity—i. e., highest luminosity—remains as a stationary point about λ 5580, midway between the Fraunhofer lines D and E. In the "Purkinje phenomenon" the maximum (visual) luminosity, in strong illumination, lies close to this point, but shifts toward the more refrangible end of the spectrum as the illumination decreases, and finally, at near the point of extinction, lies in the blue-violet. This, however, is a purely physiological phenomenon and in no wise affects the real maximum as recorded upon the photographic plate. The point of strongest action therefore remains constant.

The increase in the exposure time consequent upon the selective filtration, as compared with that of the Seed "27," was determined by equal exposures made with the Hurter and Driffield revolving sector disk. The exposures were made to diffused daylight and immediately followed one another; the entire time for both exposures being less than five minutes, the light meanwhile appearing constant. These plates were then developed together. The difference in exposure time necessary to obtain similar densities was then readily calculated from the distance apart of the plotted densities of the plates when measured, and was found to be for the Trichromatic and filter 9.2 times, and for the pinacyanol plate and filter 24.5 times. A number of subsequent exposures made in the camera confirmed these figures.

The absorption of the color-filter was next measured by the spectrophotometer in the yellow-green (λ 5500) and found to amount to 3.8 per cent.

The "actual" performance of the plate and filter in the telescope was determined by exposures upon objects of the class for which it was primarily constructed, viz., colored stars. Fig. 4, *a*, *b*, of Plate VIII shows the long-period variable *U Cygni* ($\alpha = 20^h\ 15^m\ 7^s$; $\delta = +47°\ 26'$), *B. D.* $+47°3077$, which is classified by Chandler[1] in his color-scale

[1] Third Catalogue of Variable Stars, *Astronomical Journal*, **16**, 145, 1896.

as 9.3, "where 0 corresponds to white," and so on, "through increasing shades of intensity up to the deepest red of which we have cognizance in the heavens." In the eyepiece of the 24-inch reflector this star presents an extremely beautiful appearance, almost spectacular in effect. Mean visual estimations by Messrs. Parkhurst, Jordan, and the writer, on the evening of October 3, rate it as being *possibly* a trifle brighter than its neighboring white star (*B. D.* +47°.3078), which is rated as of magnitude 8.3, and separated from it 1′ 35″. The photographic record of this intensely red star, as obtained upon the ordinary Seed "27" plate, shows it as far below its actual value; while, on the other hand, the beneficial effect of the filter and plate needs no comment.

The actual measurements of the disk diameters show that on the pinacyanol-bathed plate the red star is slightly larger compared with the white star, while on the "Trichromatic," owing to its less sensitiveness to the least refrangible end of the spectrum, the disk is of practically the same size.[1]

A further word may be said relative to the exposure through such a color-filter as has been described, and which applies generally to all others. For definitely comparable results it is essential that the temperature and time of development, and chemical constitution of the developer, be kept as constants. If this is so, then the only variable which enters into consideration is length of exposure. That this quantity must always be variable is unfortunately true, but to the worker of even limited experience the variance cannot be great, the trained observer being able to detect any decided "thickening" during the course of the exposure; this element of uncertainty obviously becomes greater as the exposure time is increased.

The influence of this variation upon the color-correction is indicated by the graduated exposures in Plate VIII, Figs. 2 and 3, and is self-explanatory. With increasing exposure up to that point which represents the true filter multiple, the remainder of the photographic opacity increases about proportionally to that at the point of maximum sensitiveness at λ 5600; beyond this point the spectrum shows a tendency to spread out at either end. This is to be expected from

[1] Further and complete information relative to this work is shortly to be published by Messrs. Parkhurst and Jordan.

the character of the filter where the absorption must be gradual and not in any way abrupt. This spreading (at the violet end) is a point, however, which need not be taken seriously, as it would require an exposure of about double the correct length of time to show any decided difference in the spectrum.

The drop in the reflectivity of the telescope mirrors as the silver films age[1] is a matter of no moment as affecting the relative exposure, as, for the work immediately under consideration, the screened exposure is always the same multiple of the unscreened plate, viz., 9.2 and 24.5 respectively. The tarnishing of the silver films ought, however, to be guarded against, as it exercises a more or less strong absorptive action upon the violet end of the spectrum according to the amount, and would thereby in critical work disturb the balance of action between the two sets of plates when taken at different periods.

It is a matter of some importance that the exposure multiple of the filter and plate over the Seed "27" be kept as nearly constant as possible, because, owing to the variance of the density-exposure curve of the plate with change in wave-length, a direct comparison of the faintest stars shown constitutes a very unreliable guide to exposure.

Although this filter has been made up primarily for use with a reflecting telescope in the work of photographic photometry, yet it will be plainly seen that its use does not end there. With the refractor it would be entirely unsuited, but with the ordinary high-grade doublet camera lens where all rays come to approximately identical focus, and on objects bright enough to allow for sufficient exposure, the gain in truthfulness of representation would be marked.

In conclusion the writer begs to acknowledge his indebtedness to Messrs. Parkhurst and Jordan for exposures at the telescope and general interest in the work.

YERKES OBSERVATORY,
October 8, 1906.

[1] C. A. Chant, *Astrophysical Journal*, **21**, 211, 1905.

ON THE BRIGHTNESS OF THE INNER EDGE OF THE PENUMBRA IN SUN-SPOTS

By S. CHEVALIER

It is commonly admitted, I think, by astronomers that in a sun-spot the inner edge of the penumbra is brighter than the outer. Father Secchi, speaking of the currents which seem to form the penumbra, says in his work on the Sun (2d ed., p. 82):

> Ces courants sont moins condensés, moins lumineux, moins nettement tranchés à l'extérieur de la pénombre, tandis que près du noyau ils se pressent, se condensent et deviennent plus brilliants. Il arrive ainsi quelquefois que le bord de la pénombre contigue au noyau acquiert un éclat plus vif, presque égal à celui de la photosphère. La tache paraît alors composée de deux anneaux brillants concentriques. Ce n'est pas une illusion due à un effet de contraste; c'est un accroissement réel de lumière, du à une condensation de matière lumineuse dans le voisinage du noyau.

Recently, Young, in his *Text-book of General Astronomy*, writes (p. 138):

> The penumbra is usually composed of "thatchstraws," or long drawn-out granules of photospheric matter, which, as has been said, converge in a general way toward the center of the spot. At the inner edge, the penumbra, from the convergence of these filaments, is usually brighter than the outer.

The contention of Faye, that this appearance is a mere illusion, could be looked on as obsolete, were it not that a most prominent authority in solar physics, Sir Norman Lockyer, revived it in his *Chemistry of the Sun*. He says (p. 408):

> To the eye the outer edge, where the half-tone is in contrast with the photosphere, seems darker than the one which is in contrast with the black nucleus. This is a subjective appearance merely; as shown in photographs, the inner edge is *not*[1] brightened.

The high authority of the writer, as well as the force presented by his argument, will very probably influence many readers. On coming across these lines, I was much surprised, and determined to try to throw a little light on the question.

But, first of all, supposing it to be true that the inner edge as shown on photographs is *not* brightened, this would not be a peremptory

[1] The italics are the author's own.

argument for calling the eye observation a subjective appearance merely. With all respect to the authority of Sir Norman Lockyer, I cannot agree to this assumption. The argument to be complete must suppose the photographs to be perfect, or at least perfect enough to show so faint a detail as a slight difference of brightness between the outer and the inner edges. Now, everyone who is acquainted with photographs of the Sun knows how many details, unmistakably perceptible to the eye in a sun-spot, do not come out on the photographic film. The small size of the picture, the unsteadiness of the image caused by the agitation of the air, the brightness of the photosphere, are some of the causes to account for this. Therefore, from the absence on photographs of a detail observed at the eyepiece, it is not fair to conclude that the eye observation was an illusion. It is first necessary to prove that this detail, if true, should have come out on the photographs.

The first remark tends to weaken the argument of Sir Norman Lockyer, but does not solve the question. To do this I have examined attentively the photographs made during the last three months at this observatory. The inner edge of the penumbra, which, as shown on the photographs at the disposal of Sir Norman, is not brightened, as shown on the photographs made at the Zô-sè Observatory, is very generally more or less brightened. This is a mere question of fact; there is no room for argument and discussion. Sir Norman has assuredly examined with attention the best photographs he had at hand, and found the inner edge *not* brightened. Regarding his assertion nobody will entertain any doubt. But I hope that the annexed page of photographs will show that on other photographs the inner edge is very generally more or less brightened. And this is enough to solve the question. Almost all the nuclear sun-spots photographed here from January 1 to April 15 are represented on this plate. Only one or two, which had been photographed in bad conditions, are not included. They are enlarged to three times the original plates.[1]

On preparing these prints we have of course paid particular attention to this phenomenon. But, besides that, the enlargements are subject by themselves to lessen the differences of brightness. The slight differences, which are undoubtedly perceptible on a negative,

[1] The effect is better shown on a set of positives sent with the manuscript.— EDS.

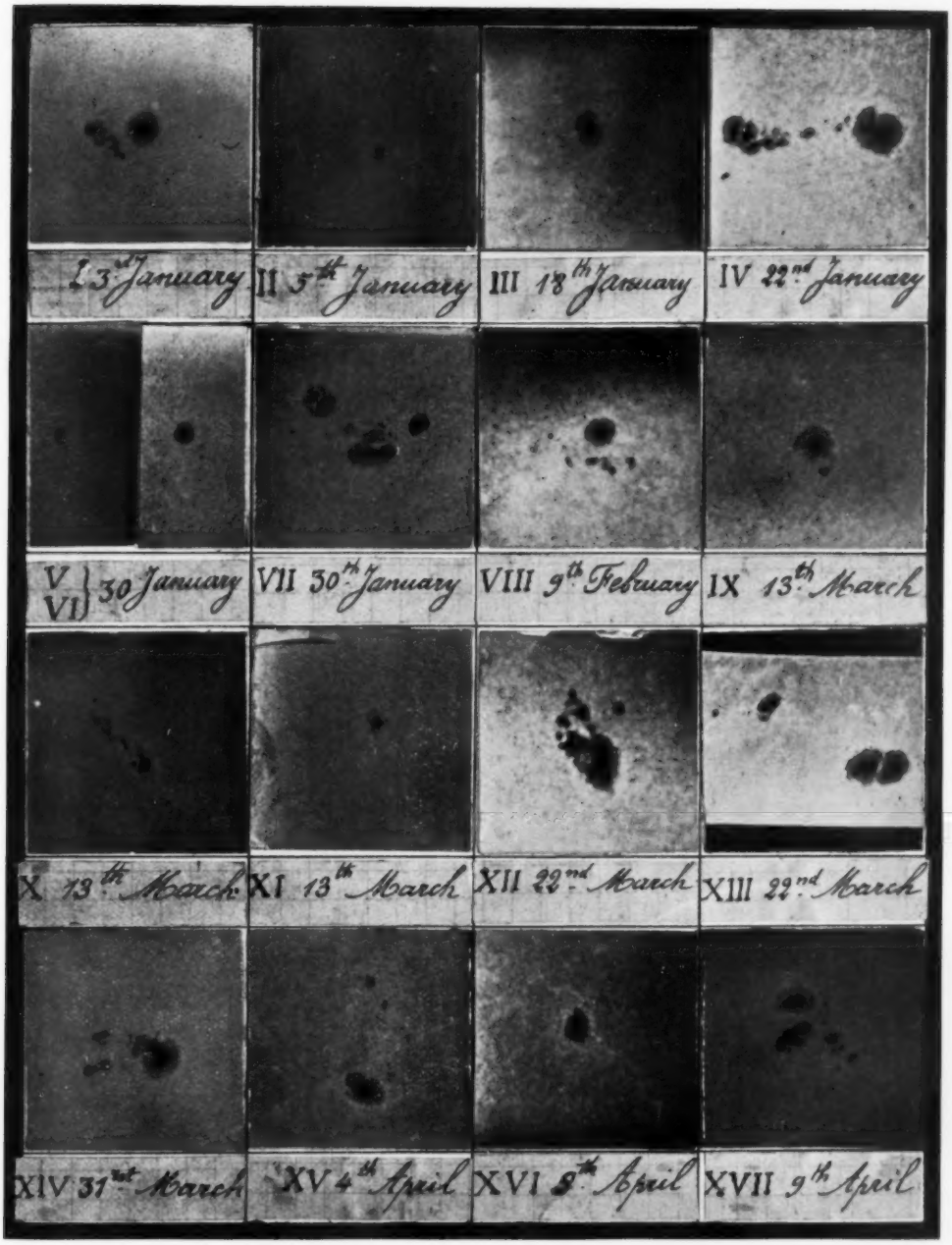
are not easily printed on paper. It is often necessary for that to sacrifice the general aspect of the picture. As in the present question the value of our photographs of the Sun will be of no slight consequence to bear conviction in the mind of the reader, I will ask the editor of the *Astrophysical Journal* to reproduce, enlarged to 3.25 times, the original negative of April 4, which I will select of course from our best photographs of this year.[1]

Now I think it will be worth while to show how far this phenomenon is perceptible on photographs, the more so that this study will lead to some interesting conclusions. For each of the seventeen sun-spots reproduced on the plate, I will tabulate the photographs obtained at this observatory, giving for each photograph, with the date, the clearness of the plate, the distance from the spot to the central meridian, and the apparent brightness of the inner edge over the outer. The Roman numerals refer to the numbers of the plate.

Date	Clearness of Plate	Dist'nce to Central Meridian	Brightness of Inner Edge
No. I. Latitude 8°.5, Longitude 282°, Surface 328 Millionths on January 3			
January 1....	Clear	1°	Very faint, almost imperceptible
" 2....	Clear	15	Well marked all round the nucleus
" 3....	Clear	29	Well marked; photographed through clouds
" 5....	Clear	54	Rather faint; well perceptible on NNE and S of the nucleus
" 6....	Clear	67	Well perceptible NE and SE of the nucleus
No. II. Latitude 10°.0, Longitude 178°, Surface 63 Millionths. January 5			
January 5....	Clear	−49°	Strongly marked
" 6....	Clear	−37	Strongly marked
" 7....	Clear	−23	The penumbra is fading away
No. III. Latitude 8°.5, Longitude 104°, Surface 320 Millionths. January 9			
January 9....	Very clear	−71°	Very strong except on W side
" 12....	Foggy	−32	Very faint, almost imperceptible
" 13....	Very clear	−18	Strongly marked
" 14....	Clear	− 5	Strongly marked
" 15....	Clear	8	Well marked, but not so strong
" 16....	Clear	21	Well marked
" 17....	Clear	34	Well marked
" 18....	Very clear	47	Very strongly marked
" 20....	Foggy	75	Strongly marked on the N side

[1] Although unable to comply with the author's request, we are prepared to vouch for the excellence of this photograph.—EDS.

PLATE X



Brightness of Inner Edge of Penumbra in Sun-Spots

Date	Clearness of Plate	Dist 'nce to Central Meridian	Brightness of the Inner Edge
No. IV. Latitude −11°.0, Longitude 336°, Surface 377 Millionths. (Largest nucleus)			
January 25....	Very clear	10°	Faint, except on few places
" 26....	Very clear	25	Easily perceptible
" 27....	Rather clear	40	Faint
" 28....	Rather clear	53	Faint
" 29....	Clear	65	Well marked round the first nucleus
" 30....	Very clear	78	Clearly visible round the three nuclei
No. V. Latitude 15°.9, Longitude 302°, Surface 118 Millionths. January 29			
January 27....	Rather clear	6°	Faint, the penumbra is just making its first appearance
" 28....	Rather clear	18	Well marked
" 29....	Clear	31	Well marked on the W side
" 30....	Very clear	44	Well marked
No. VI. Latitude −7°.4, Longitude 286°, Surface 114 Millionths. January 28			
January 25....	Very clear	−40°	Well marked
" 26....	Clear	−26	Strongly marked
" 27....	Rather clear	−10	Faint
" 28....	Rather clear	1	Well marked
" 29....	Clear	14	Rather faint
" 30....	Very clear	28	Strongly marked
February 2....	Faint	70	Not clear
No. VII. Latitude 17°.0, Longitude 234°, Surface 297 Millionths. (Largest Spot of the Group)			
January 28 ...	Rather clear	−51°	Well marked
" 29....	Clear	−38	Visible only W of the second nucleus
" 30....	Very clear	−25	Faint round the first nucleus, well marked round the second
February 2....	Faint	18	Very faint
" 3....	Foggy	30	Very faint
" 4....	Foggy	42	Very faint
" 5....	Clear, faint	55	Well marked
No. VIII. Latitude 9°.5, Longitude 105°, Surface 175 Millionths. February 9			
February 5....	Clear, faint	−75°	Very faint
" 6....	Foggy	−59	Very faint
" 7....	Rather clear	−48	Well marked
" 8....	Tolerably clear	−31	Faint, well visible
" 9....	Clear	−21	Well marked
" 11....	Clear	8	Well marked

Date	Clearness of Plate	Distance to Central Meridian	Brightness of Inner Edge
No. IX. Latitude 5°.5, Longitude 74°, Surface 227 Millionths. March 12			
March 8....	Clear, faint	−60°	Well marked
" 9....	Clear	−47	Strongly marked
" 12....	Clear	− 5	Strongly marked
" 13....	Very clear	8	Very strongly marked
" 14....	Faint, soft	22	Very faint
" 15....	Very clear	36	Strongly marked
" 16....	Clear	50	Strongly marked
" 17....	Clear	67	Strongly marked on the N, and slightly on the W
No. X. Latitude 19°.5, Longitude 107, Surface 411 Millionths. March 15			
March 12....	Clear	27°	Strongly marked S and W
" 13....	Very clear	40	Well marked
" 14....	Tolerably clear	55	Faint
" 15....	Clear	70	Well marked on N and S
" 16....	Clear	84	Too near the limb
No. XI. Latitude 6°.0, Longitude 88°, Surface 100 Millionths. March 12			
March 12....	Clear	8°	Rather well marked
" 13....	Very clear	21	Well marked
" 14....	Tolerably clear	37	Faint
No. XII. Latitude 20°.0, Longitude 313°, Surface 350 Millionths. March 21			
March 18....	Foggy	−45°	Invisible
" 19....	Clear	−34	Easily visible
" 21....	Clear	− 2	Irregularly marked round the nucleus
" 22....	Very clear	11	
No. XIII. Latitude 14°.4, Longitude 273°, Surface 414 Millionths. March 19			
March 19....	Clear	−75°	Very faint
" 21....	Clear	−43	Strongly marked, photographed through clouds
" 22....	Very clear	−30	Strongly marked
" 30....	Clear	74	Well marked N and S of the nucleus
No. XIV. Latitude 8°.3, Longitude 243°, Surface 199 Millionths. March 21			
March 21....	Clear	−74°	Spot too faint
" 22....	Very clear		Well marked
" 30....	Clear	43	Well marked
" 31....	Clear	57	Well marked

Date	Clearness of Plate	Dist.'nce to Central Meridian	Brightness of Inner Edge
No. XV. Latitude −12°.8, Longitude 174°, Surface 427 Millionths. March 31			
March 30....	Clear	−32°	Well marked on the S
" 31....	Clear	−17	Faint
April 1....	Foggy	− 3	Faint
" 2....	Clear	11	Well marked
" 3....	Rather clear	24	Well marked
" 4....	Very clear	37	Strongly marked
" 6....	Clear	65	Strongly marked N, S and W
" 7....	Clear	78	Too near the limb
No. XVI. Latitude 20°.0, Longitude 106°, Surface 312 Millionths. April 1			
April 1....	Foggy	−69°	Well marked
" 2....	Clear	−55	Strongly marked N, S and E of the nucleus
" 3....	Rather clear	−42	Very strongly marked
" 4....	Very clear	−30	Very strongly marked
" 5....	Clear	− 4	Strongly marked
" 7....	Clear	9	Strongly marked
" 9....	Clear	38	Strongly marked
" 10....	Clear	49	Well marked
No. XVII. Latitude 26°.0, Longitude 43°, Surface 314 Millionths. April 7			
April 6....	Clear	−66°	Too near the limb
" 7....	Clear	−53	Well marked on N, S and E
" 9....	Clear	−25	Faint
" 10....	Clear	−15	Rather well marked
" 11....	Clear	− 2	Well marked
" 12....	Foggy	12	Well marked

From these tables it seems possible to derive the following conclusions:

1. The brightness of the inner edge of the penumbra in a sun-spot is a very common phenomenon, undoubtedly true.

2. There is a great difference with regard to this point, from one sun-spot to another.

3. For each sun-spot there is not any apparent variation in this phenomenon, resulting from its place over the disk. It must, however, be noted that the first appearance of it comes out on the north and south of the nucleus, when the spot is entering on the visible hemisphere, then on the east, and finally on the west side. When the spot is approaching the west limb, the disappearance begins with the

east side, then the west, and finally the north and south. It is just the same as for the penumbra itself. This remark, though of a negative character, is of no small importance, as it shows a wide difference between the brightness of the inner edge of the penumbra and the brightness of the faculæ.

4. This brightness of the inner edge seems to be one of the characteristic notes of a sun-spot, subject, very probably, to some real but only slight changes.

5. There is a strong probability that the sun-spots of a regular shape present this phenomenon with a more marked brilliancy.

ZÔ-SÈ OBSERVATORY,
Near Zi-Ka-Wei, Shanghai, China.

THE SPECTROCOMPARATOR[1]

By J. HARTMANN

In the *Publicationen des astrophysikalischen Observatoriums zu Potsdam*, Bd. **18,** Stück 53, 1906, which has recently appeared, I have communicated a procedure for the measurement of lines in spectrograms which is based on the application of a special measuring apparatus. In the following I will give the most important points concerning the purpose of this instrument, its construction, and its application.

Hitherto the method of measurement of the Doppler displacement of lines in stellar spectra consisted in setting the cross-wires of a measuring microscope upon one after another of the lines of the stellar spectrum and the adjacent terrestrial comparison spectrum, generally that of iron. In order to compute the displacements of the lines from the screw readings and from these the value of the radial velocity of the star, the wave-lengths of the lines must be assumed as known. In the case of the first-type stars, where there is a paucity of lines, this method has so fully sufficed that a better substitute is scarcely to be made. For the few lines of these stellar spectra can be measured in their entirety without great expenditure of time and their wave-lengths, belonging as they do to a few gases, principally hydrogen and helium, are known with great precision. The complete measurement and reduction of such a spectrum, when the convenient method is employed which I described in the *Astronomische Nachrichten*, **155,** 81, 1901, requires about two hours.

The conditions are quite different in spectra of the later star types, where there are many lines. In the first place, the lines in these spectra are so numerous that their complete measurement and reduction would require many days, and in the second place a rigorous reduction of such a material has hitherto not been at all possible because the wave-lengths of the lines are not known with sufficient accuracy. On this account, observers have until now limited them-

[1] Translated by Philip Fox from the *Zeitschrift für Instrumentenkunde*, **21**, 205–7, July 1906. The electrotypes have been kindly loaned by Director Vogel of the Astrophysical Observatory at Potsdam.

selves to a partial treatment of such spectra, measuring only a small number of lines, whereby the major part of the rich material present in the plate remains unutilized. In addition it must be noted that the value of the radial velocity derived from these measurements must undergo a slight correction when definitive values of the wave-lengths used become known. The difficulty of ascertaining the correct wave-lengths for these star spectra lies in part in the fact that the stellar spectrographs used for the exposures are not sufficiently powerful to separate fully the lines lying near together, so that these very often merge together. For the truly isolated lines exact wave-lengths are also lacking. The observer must take them from Rowland's table of the solar spectrum, the wave-lengths of which, up to the present, have not been adjusted to the laboratory iron spectrum in a manner free from objection, so that the systematic differences in the two systems of wave-lengths must enter into and falsify all spectroscopic measurements of velocity. In addition the truly isolated lines are frequently so fine as to be hard to set upon with the cross-wire; and further it is never possible until the end of the whole computation to make certain, in a way free from arbitrariness, from the accordance of the different values found for the velocity, whether or not decidedly erroneous wave-lengths have been assumed for numerous lines.

All of these difficulties are obviated by the new method of measurement which makes possible the complete measurement and reduction of a spectrum, however rich in lines, and the derivation of the definitive velocity in from one to two hours. This great advantage is reached not by setting as formerly the cross-wire upon single lines, but by simultaneously bringing into coincidence numerous lines of one spectrum and the corresponding lines of another.

The exposures of the spectrograms with the spectrograph are made as formerly, the terrestrial comparison spectrum, generally iron, appearing on both sides of the star spectrum. In the plates exposed with the Potsdam Spectrograph III, the stellar spectrum is 0.25 mm wide and on both sides lie the iron spectra 0.60 mm wide, the intervals between the stellar- and iron-spectra being 0.25 mm. With the same spectrograph a plate of the solar spectrum is made which should have a width of 0.55 mm, thus leaving intervals 0.1 mm

wide. The exact widths may be easily secured with the help of my occulting device.

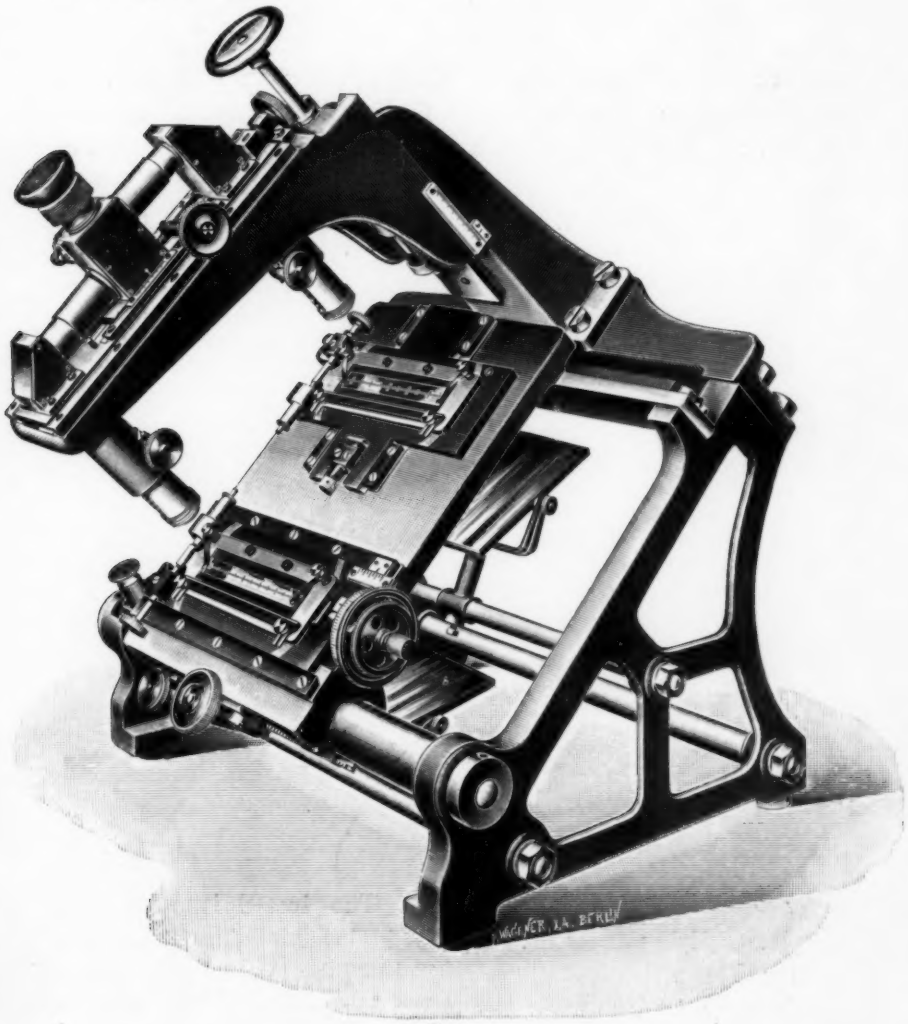


FIG. 1

The measuring apparatus was constructed according to my designs by the firm of Carl Zeiss in Jena, under the especial direction of Dr. Pulfrich. Fig. 1 shows the general appearance of the instrument, which I now proceed to describe.

The stage of the measuring apparatus, on which are clamped the two spectra to be compared, is shown in Fig. 2 in one-fifth its natural size. It holds the device for the orientation of the two plates. The solar spectrum mentioned above is attached by means of two microscope clamps to plate A_1, which has an aperture 1 cm wide and 12 cm long for the illumination of the spectrum. The plate A_1 can be turned about a short central pivot E_1, by means of the

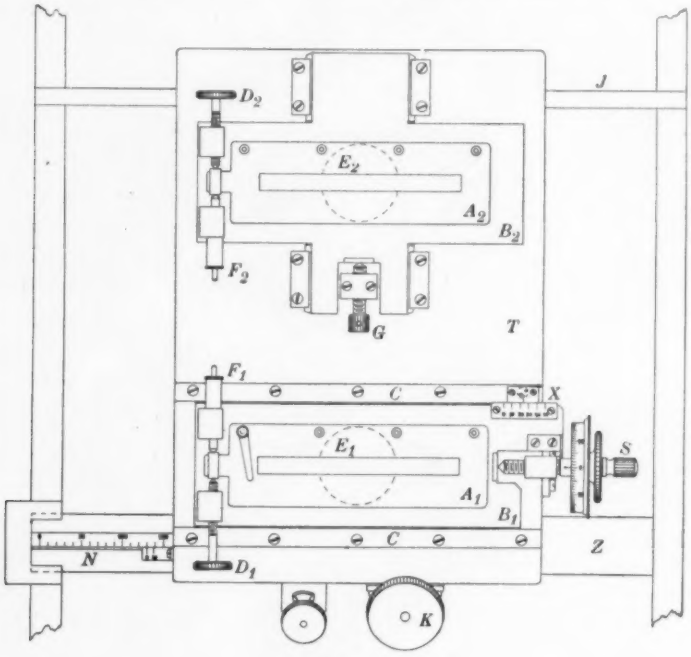


FIG. 2

screw D_1 and the opposing spring F_1. The pivot E_1 is mounted on the plate B_1 lying under A_1. B_1 runs in dovetail guides C, and can be moved from right to left on the Table T by means of the micrometric screw S. The screw has a pitch of 0.5 mm, and its head has 100 divisions, so that the displacement of the spectrum from right to left can be read to 0.0005 mm. The screw runs 45 turns in its nut and is usable for somewhat over 2 cm. Two long spiral springs lying underneath the table press the plate B_1 with its agate buffer against the rounded end of the screw. The number of complete

turns of the screw is read on the scale X which has half-millimeter divisions.

The stellar spectrum to be measured is mounted in similar fashion on the plate A_2, which may be turned about the pivot E_2. Plate

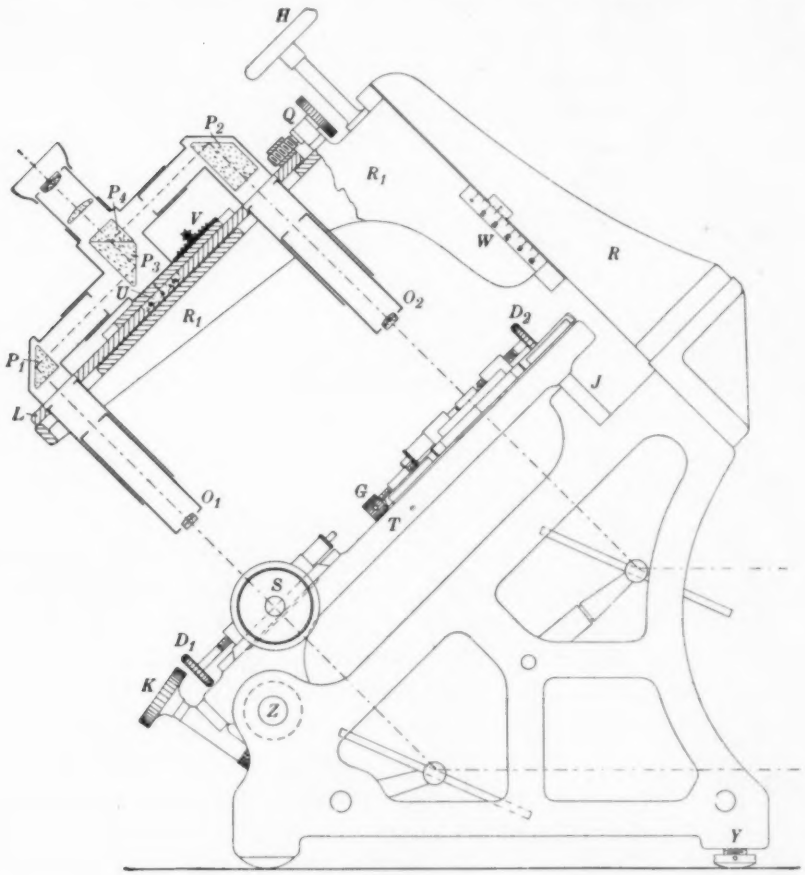


FIG. 3

B_2, carrying A_2, may be moved up and down on the table T by means of the screw G, thus enabling the distance between the two spectra to be regulated.

The whole table T is movable from left to right on the steel cylinder Z, 35 mm in diameter, and the steel guide J by means of the pinion K meshing in a rack lying underneath. Near this is

seen a clamp for holding the table in an invariable position. The setting can be read on the scale N, graduated in half-millimeters, by means of a vernier and magnifying lens.

The stage T, as is apparent[1] in Fig. 3, is carried by its guides Z and J in a plane inclined 45°. Above it stands the carrier of the double microscope RR_1, the construction of which is manifest in Fig. 3. The optical axes of the two objectives O_1 and O_2 are perpendicular to the two spectrograms clamped to the table below. The two objective tubes, each provided with draw tubes 41 mm long and adjustable with rack and pinion, are fastened on the plate L at a fixed distance from one another. The plate L may be moved about 1 cm in its dovetail guides on the surface of the carrier R_1, by means of the screw Q. At the upper ends of the two objective tubes the beams of light are reflected toward the prism-system P_3P_4 by the right-angled prisms P_1 and P_2. At the surface of contact of the two prisms P_3 and P_4 the union of the two beams takes place in the following manner.

On the hypothenuse surface of the prism P_4 the portion shown shaded in Fig. 4 is silvered, and then the prism P_4 is cemented to the prism P_3. In the eyepiece the field of view has the form shown within the circle. If the two spectra are laid under the microscopes O_1 and O_2 and properly adjusted, the observer perceives, as shown in Figs. 5 and 6, part of the stellar spectrum or its comparison spectra on the shaded portion of the surface, while the remainder of the field is filled with the image of the solar spectrum and its comparison spectra lying under O_1.

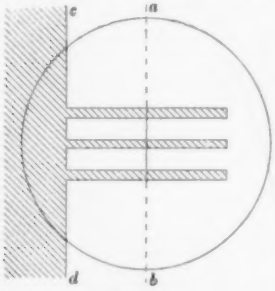


FIG. 4

Through the middle of the hypothenuse surface of the prism P_4 a fine black line ab is drawn perpendicular to the strips of mirror. It serves not for real measurement but merely to roughly mark the middle of the field of view. I will call it simply the cross-wire, in analogy to the measuring cross-wire of other microscopes. Since this cross-wire is inclined 45° to the optical axis of the eyepiece, only a short stretch of it will appear

[1] In the schematic Fig. 3 all screws are omitted for the sake of simplicity.

sharp in the field of view at any time. In like manner the edges of the two outer strips of the mirror will appear somewhat out of focus if the eyepiece is set carefully upon the middle strip. In the measurements this is entirely immaterial, for the observer does not see the mirror at all when the plates are properly illuminated, but sees only the images reflected from them, and these lie accurately in the focal plane.

The prism-system P_3P_4 is mounted in a small box, from which it may be easily removed in order to substitute another. By this means it is possible to use different forms of mirror as desired. The apparatus has a prism-system additional to that described above, which serves for bisecting the field of view, so that the entire lower half of the field is filled by the image of the plate lying under O_1, and the upper half by the image of the plate under O_2.

The small box holding the prism-system and also carrying the eyepiece is mounted on a slide which may be moved by means of the rack at V, the positions being read on the millimeter scale U. If the box approaches the microscope O_1, this microscope is shortened and the magnification diminished, while at the same time the magnification of O_2 is increased. By this gradual variation of the relative magnification of the two microscopes the observer can make the two images of the spectrograms in the field of view so exactly of a size that they can be brought simultaneously into sharp coincidence throughout their entire extent. This arrangement for the establishment of images of exactly equal size, applied here so far as I know for the first time, is of fundamental importance in the entire process of measuring.

Finally it is to be mentioned that the upper part of the microscope carrier R, may be moved about 5 cm up and down in the dovetail guides on the lower part R, by the use of the hand wheel H. This motion allows the magnification of the microscopes to be changed simultaneously, so that the width of the images of the spectra may be made to agree with the given width of the mirror in the prism-system; that is, the two outer strips are made to lie in the centers of the comparison spectra. The height of the microscopes is read on the millimeter scale W. If this motion, which allows a variation of 30 per cent. in the size of the image, does not suffice for adapt-

ing the width of a given spectrum to the distance of the mirror-strips, the observer can substitute another pair of objectives of suitable focal length.

The entire frame of the apparatus is sufficiently stable despite its light and pleasing form. It rests upon four feet, of which the one at Y is adjustable by a screw, and in the usual way, by testing opposite corners, any unevenness of the bed may be accommodated so that the instrument stands on its four feet free of strain.

Since the measuring makes no high demands on the mechanical construction, the testing is very simple. It is never assumed in the measurement that the motions are exactly in a straight line or perpendicular to one another, and consequently this investigation can be omitted. Only the optical parts and the measuring screw must undergo a close test. Since deterioration of the images may result from the passage of light through the prisms and from the two reflections, the observer assures himself against this by the perfect sharpness of the images when observing a fine test object. Further, the three strips of the mirror in the prism-system should be sharp and the borders approximately parallel to each other and to the cylindrical guide Z. This can be tested by bringing a point of the image of a spectrum, clamped in position, upon the edge of a mirror-strip, when upon moving the table the point should follow along the edge of the strip.

I will pass over the investigation of the screw, which was carried out in the usual way, simply remarking that the screw of the Potsdam instrument is especially good. In the stretch of 35 revolutions which I investigated neither the periodic nor the progressive errors ever amounted to 0.001 rev. $=0.0005$ mm. For the 8 middle revolutions which alone are used in the measurements the periodic error is

$$-0.0001285\ \text{Rev.}\ \cos A + 0.0003484\ \text{Rev.}\ \sin A\ .$$

Though the maximum of this is only 0.00037 rev. $=0.000185$ mm, it may not be neglected in the measurement of stellar spectra, for in unfavorable cases it may cause an error of 0.27 km in the determination of the velocity, while a precision of 0.1 km is now attainable in these measurements. I shall show farther on how the slight influence of the periodic errors can be made harmless by proper arrangement

of the measurements. The progressive errors need not be considered, for in the measurement only fractions of a revolution are used

In order to simplify the measurements, the observer determines a few constants of the instrument, viz., the middle points of the scales N, X, and U, and constructs a table for the elevation W.

I call the central setting, N_0, of the table, that when the image of the center of rotation of the pivot E_2 is on the cross-wire. Clamp the table in this position and move the slide B_2 by means of the screw until the center of rotation of E_1 lies on the cross-wire. This gives the reading X_0.

The scale U is an index to the setting of the relative magnification of the two microscopes, or, as I call it briefly, the image-extension (*Bilddehnung*). To find the position U_0 in which the two magnifications are equal, lay under the microscopes any two plates which have on them a sharp mark and bring the images into coincidence by turning the measuring screw. If the table T is moved back and forth on the cylindrical guide, the marks remain in coincidence in the entire field of view only when the magnifications of the two microscopes are equal. If the mark lying under O_1 moves less than the other, then the image from O_1 is less magnified, the microscope O_1 is too short, and the prism-system is therefore to be moved upward in the positive direction of the scale U, by means of the rack V. By this motion the sharp setting on the images is somewhat disturbed, and the observer must correct by refocusing the two objectives. To simplify this, which frequently recurs in the measurement of spectra, I have so arranged the racks of the draw-tubes that to attain a sharp setting, the pinions must be turned in the same direction as the pinion V was previously turned. The observer should mark once for all the direction in which V must be moved: when the image of the spectrum lying under O_1 (later the solar spectrum) is too *small*, the motion of the prism-system is then *upwards*.

As a final simplification for the coming measurements, the observer can form a small table which gives the setting of the elevation-scale W for various widths of spectra. The microscope-carrier R_1 must always be set so that the two outer strips of mirror lie in the centers of the iron spectra. In order to find this setting quickly by means of the scale W one needs only to know the size of object this mirror

distance represents at various readings of W. Measure then the distance between the centers of the comparison spectra and set W accordingly.

To construct this table lay a scale of 0.1 mm divisions, having about the thickness of the spectrograms, on the plate A_1 so that the divisions lie horizontally. Keeping the "image-extension" constant at U_0, make a series of settings on the scale W, focus for each setting with the objective draw-tube and estimate on the scale the distance between the centers of the two outer strips.

Two pairs of objectives belong to the Potsdam instrument. They are of 55 mm and 45 mm focal length and I designate them I and II. Further, since the objective draw-tubes could not be made long enough to use the whole of the scale W, I have had a pair of extension tubes, 20 mm long, made which may be screwed into the draw-tubes. With the combinations now possible the following values $2D$ were obtained from measurements of the mirror distance on the scale:

Objective	W	$2D$	p_2	p
I without extension tube.....	6	1.57 mm	2.54	21.2
	10	1.53	2.60	21.7
	15	1.47	2.71	22.6
	20	1.42	2.81	23.4
	25	1.37	2.91	24.3
I with extension tube.....	30	1.33	3.01	25.1
	35	1.28	3.11	25.9
	40	1.24	3.21	26.8
	45	1.21	3.30	27.5
	50	1.17	3.40	28.4
	55	1.14	3.50	29.2
II without extension tube.....	0	1.10	3.62	30.2
	5	1.06	3.76	31.4
	10	1.03	3.87	32.3
	15	1.00	3.98	33.2
	20	0.97	4.11	34.3
	25	0.94	4.24	35.4
II with extension tube.....	30	0.91	4.38	36.5
	35	0.89	4.48	37.4
	40	0.87	4.58	38.2
	45	0.85	4.69	39.1
	50	0.83	4.80	40.0
	54	0.81	4.92	41.0

It is seen that the mirror can be set sharply in the middle of the comparison spectra when the distance between them lies between 0.81 mm and 1.57 mm. All the exposures with the Potsdam Spectrograph III lie within this range.

The value $2D$ offers each time an easy means of computing the magnification. Since a knowledge of this is also of interest I will include a determination of it. In order to find the objective magnification p_1, divide the interval between the outer mirror strips, as projected in the focal plane, by $2D$. I have measured this distance microscopically and found it to be 3.982 mm. From this is obtained the objective magnification given in the table under p_1.

The magnification of the eyepiece cannot be readily computed directly from its focal length, for the beam of light passes not through air, but almost entirely in the glass of the prism-system behind the eyepiece. I have therefore measured directly outside the eyepiece, the angle of divergence 2ϕ between the two rays coming from the two outer mirror-strips. This gives $2\phi = 7^\circ.6$. From this follows the eyepiece magnification referred to a distance of vision of 25 cm

$$p_2 = \frac{50 \tan \phi}{3.982} = 8.34 ,$$

and the combined magnification given in the table

$$p = p_1 p_2 .$$

If for cases in which the enlargement (which may be brought up to 41 diameters) appears too great, there is included a second eyepiece for the apparatus which gives half the above magnifications.

I shall now give an illustration of the use of the apparatus, treating it briefly and referring for details to the more exhaustive communication mentioned in the beginning.

The solar spectrum placed under the microscope in the measurement of stellar spectra I shall call the fundamental spectrum because all velocities measured depend upon it. With the comparator the difference of the displacements of the lines is directly measured; and therefore the difference of velocities as given by the stellar spectrum and fundamental spectrum. Since the observer can compute the radial velocity of the Sun, the velocity of the star is at once found. I will only mention here further that the spectrum of any star can be used in place of that of the Sun as the fundamental spectrum, but for the sake of simplicity I shall speak only of the solar spectrum,

On the solar spectrum plate very near the iron spectrum the observer marks a number of places by means of dots, and numbers

them as shown in Figs. 5 and 6. The places are so chosen that when the dot is upon the cross-wire, a number of good lines of the iron spectrum are on the mirror-strips on both sides of the cross-wire. For these regions the observer computes the velocity-factors, s, that is, the velocity which 1 rev. of the screw represents for each region. These values are conveniently written upon the right edge of a card upon which appears also the value of W and of V_o, and a small table of the values of j, of which I am about to speak. The card for the fundamental spectrum III 758 has the following form:

FUNDAMENTAL SPECTRUM III 758
SUN, 1905 MAY 16.
$W=30.0$; $V_o=+0.31$ km

	12	13	14	15	16
23	1.23569	1.26718	1.30207	1.34125	1.38584
24	1.19342	1.22188	1.25319	1.28803	1.32724
25	1.15366	1.17956	1.20787	1.23912	1.27399
26	1.11600	1.13969	1.16544	1.19369	1.22496
27	1.08024	1.10201	1.12556	1.15126	1.17952
28	1.04614	1.06623	1.08786	1.11136	1.13707
29	1.01354	1.03214	1.05211	1.07369	1.09721
30	0.98216	0.99944	1.01793	1.03785	1.05945
31	0.95191	0.96801	0.98518	1.00362	1.02355
32	0.92292	0.93796	0.95396	0.97111	0.98956
33	0.89496	0.90904	0.92399	0.93997	0.95713
34	0.86790	0.88112	0.89513	0.91006	0.92605

12	492
13	479
14	465
15	450
16	437
17	422
18	407
19	396
20	383
21	373
22	361
23	349
24	337
25	326
26	315
27	304
28	294
29	285
30	275
31	266
32	259
33	252
34	244

The measuring of the stellar spectrum is done as follows: The observer sets for the value of W given on the card and for the image-extension, U_o. If X is the accidental reading of the measuring screw, set the table at the reading $N=N_o-X_o+X$; at this point the axis of the pivot E_1 is on the cross-wire. The fundamental spectrum is then clamped on the plate A_1 in position I, by which I denotes that with the longer wave-lengths to the right, or in which they appear to the left in the microscope. The observer sets the eyepiece sharply on the middle mirror-strip and brings the image of the plate in sharp coincidence with it by means of the draw-tube O_1. Using the screw Q the middle strip is set exactly in the middle of the

solar spectrum, the table is moved to the end of the spectrum by means of the pinion K, and by the screw D_1 the spectrum is reset until it is again symmetrical to the mirror. The fundamental spectrum is then parallel to the cylindrical guide and its adjustment is complete, a process requiring less than a minute. When the instrument is in continuous use the observer will generally allow the fundamental spectrum to lie undisturbed for a long time and the adjustment described above is eliminated.

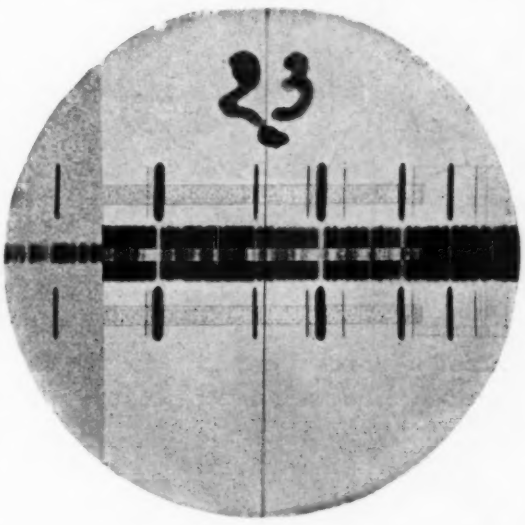


FIG. 5

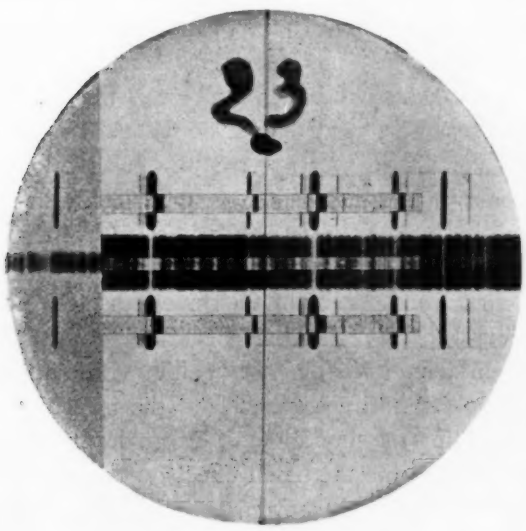


FIG. 6

The adjustment of the star spectrum is almost equally simple. It is clamped on the plate A_2 with the same region of the spectrum in the field of view as with the fundamental spectrum. The objective O_2 is set carefully, the table is brought to its central position N_0, and the stellar spectrum is moved by the screw G until it appears at an equal height in the field of view with

the fundamental spectrum. This adjustment is made with the iron lines of the two plates alone and they are compared near the line of separation *cd* in Fig. 4. The table is moved again sidewise to the end of the spectrum and the stellar spectrum is again set at an equal height by means of the screw D_2. The stellar spectrum also is now parallel to the cylindrical guide.

Then the "image-extension" is to be so adjusted by the rack *V* that the two spectra in the field of view appear of equal size. By observing the rule given above, concerning the direction in which *V* is to be turned, this, too, is easily accomplished.

If the stellar spectrum, as is frequently the case, is not uniformly strong throughout its width, the observer moves the microscopes by means of the screw *Q* until the middle strip of mirror falls upon the best part of the stellar-spectrum, and with this, all the preparations are ended.

The real measurement consists essentially, while one of the various numbered points is on the cross-wire, in bringing first, the stellar spectrum into coincidence with the adjacent solar spectrum, and then the two iron spectra into coincidence, as may be seen from Figs. 5 and 6. Since the spectra exactly coincide throughout the length of the field of view, the observer recognizes at a glance which lines are common to both spectra and can be used in establishing the coincidence. For this the lines need not be sharply defined nor symmetrical, but wide groups, edges of bands, and even the dark intervals of the negative between lines are equally applicable. On account of these continual comparisons of spectra, the observation is very interesting, and moreover is so rapid that the complete measurement of a spectrum rich in lines is accomplished in about a half-hour. The number of the coincidences of lines actually observed in this time is very great, since each setting of the screw depends on very numerous simultaneously observed coincidences.

When the spectrum is measured in the first position, both the stellar spectrum and solar spectrum are reversed and the measurement is repeated, thus eliminating the physiological errors dependent upon the position. At the same time the small effect on the result of the periodic screw errors mentioned above is eliminated. For this purpose the observer has only to heed that the measurement in

the second position is made at a place in the screw 180° from that in the first.

The reduction is as simple as the measurement, as I will illustrate with an example. The following scheme contains all the computations which can present themselves in any case, but under circumstances about half of it may be omitted. I remark further that mark No. 13 of the fundamental spectrum is at λ 4606, mark No. 34 at λ 4070.

The four columns designated *Fe* and * give the screw-readings taken in the measurement. The amount of the line-displacement is deduced from them, viz.:

$$\text{in position I, } d_1 = Fe - *,$$
$$\text{in position II, } d_2 = * - Fe.$$

The column $d_2 - d_1$ is computed only as a check. Negative values of $d_2 - d_1$ are seen to predominate. This is common to all series and arises from the above-mentioned physiological difference of perception dependent on the position of the spectrum. Its influence on the result is eliminated by forming the mean d from the d_1 and d_2. The computer now places the card described on p. 296 so that the values of s belonging to the various regions stand opposite the corresponding values of d; then with the help of a multiplication table he forms the product $sd = V_* - V_\circ$. These are the values of the difference in velocity between the star and the fundamental spectrum, of which the mean M_1 in our case is +11.60 km.

The reduction here described is so simple that it can be made in a few minutes, and it is so easy to look over that large blunders are scarcely possible. However I use this computation only as a control and deduce the definitive value of the velocity in another still more simple and more accurate way.

In obtaining the mean value M_1, the individual differences of velocity $V_* - V_\circ$ from the various parts of the plate are given equal weight. This is not rigorously correct on account of the strongly increased dispersion toward the violet end of the spectrum. Since one can suppose that on the average the linear amount d of the line-displacement can be measured throughout with the same precision, the values $V_* - V_\circ = sd$ should receive, in the formation of the mean,

PLATE III 775. α *Boötis* 1905 MAY 26, $9^h\,44^m$ C. E. T.
FUNDAMENTAL SPECTRUM III 758.

MARK No.	FIRST POSITION			SECOND POSITION			d_2-d_1	d	V_*-V_o	F
	Fe	*	d_1	*Fe*	*	d_2				
	Rev.	Rev.	Rev.	Rev.	Rev.	Rev.	Rev.	Rev.	km	km
13	27.403	0.382	+0.021	27.876	0.901	+0.025	+0.004	+0.023	+11.0	−0.6
14	.406	.380	26	.875	.899	24	− 2	25	11.6	0.0
15	.404	.377	27	.876	.900	24	− 3	25_5	11.5	−0.1
16	.408	.376	32	.869	.898	29	− 3	30_5	13.3	+1.7
17	.410	.380	30	.873	.902	29	− 1	29_5	12.4	+0.8
18	.413	.385	28	.870	.900	30	+ 2	29	11.8	+0.2
19	.423	.395	28	.862	.891	29	+ 1	28_5	11.3	−0.3
20	.424	.392	32	.860	.886	26	− 6	29	11.1	−0.5
21	.427	.389	38	.856	.888	32	− 6	35	13.1	+1.5
22	.427	.398	29	.856	.889	33	+ 4	31	11.2	−0.4
23	.426	.393	33	.853	.882	29	− 4	31	10.8	−0.8
24	.427	.392	35	.852	.885	33	− 2	34	11.5	−0.1
25	.427	.388	39	.857	.889	32	− 7	35_5	11.6	0.0
26	.423	.390	33	.857	.888	31	− 2	32	10.1	−1.5
27	.426	.385	41	.855	.893	38	− 3	39_5	12.0	+0.4
28	.423	.380	43	.858	.893	35	− 8	39	11.5	−0.1
29	.424	.386	38	.857	.892	35	− 3	36_5	10.4	−1.2
30	.426	.380	46	.856	.900	44	− 2	45	12.4	+0.8
31	.421	.373	48	.856	.905	49	+ 1	48_5	12.9	+1.3
32	.421	.378	43	.859	.900	41	− 2	42	10.9	−0.7
33	.424	.380	44	.855	.898	43	− 1	43_5	11.0	−0.6
34	.430	.377	53	.852	.895	43	− 10	48	11.7	+0.1
		Σd_1	+0.787		Σd_2	+0.734		M_1	+11.60	
			+0.734							
			+1.521							

$\log\ \ =0.18213$ $\qquad M_2=+11.57$ km
$\log f\ =0.88112$ $\qquad V_o=+\ 0.31$
$\log M_2=1.06325$ $\qquad V=+11.88$ km

weights which are proportional to $\frac{1}{s}$. Designate the mean so calculated with M_2, then

$$M_2=\frac{\Sigma\frac{1}{s}(V_*-V_o)}{\Sigma\frac{1}{s}}=\frac{\Sigma d}{\Sigma\frac{1}{s}}.$$

This correct manner of reckoning thus leads to an extraordinarily simple scheme of computation. It is unnecessary to form the individual products, *sd*, but only to multiply the sum of all the *d's* with a

factor which is constant so long as the same regions are used. Since

$$\Sigma d = \frac{1}{2}(\Sigma d_1 + \Sigma d_2),$$

I set

$$f = \frac{1}{2\Sigma\frac{1}{s}},$$

and from this comes

$$M_2 = f(\Sigma d_1 + \Sigma d_2).$$

The values of log f are given on the card described above for the first and last regions employed as arguments. In our illustration we measured from mark No. 13 to No. 34, whence we have log f = 0.88112 and from this, as given under the scheme of reduction, $M_2 = +11.57$ km, which agrees closely with M_1. After adding to M_2 the value of V_0, also given on the card, the velocity sought, of the star relative to the Earth, is obtained,

$$V = +11.88 \text{ km}.$$

This can be referred to the Sun in the usual way.

As seen in the example the entire measurement and computation is extraordinarily simple, so that the complete treatment of a stellar spectrum, which formerly would have required many days, now needs at most two hours. In addition to this, the new method is perfectly rigorous, since no inadmissible suppositions were made and nothing was omitted.

On account of the extraordinarily great number of coincidences used, the accuracy of the new method is greater than in the earlier method; but more especially on account of the elimination of all errors which were formerly introduced into the result from the adoption of incorrect wave-lengths of the lines observed the value of the velocity is considerably more certain and is definitive. In the paper cited in the beginning of this article I have made a more thorough investigation of the accuracy of the method from the standpoint of a greater quantity of observational material, of which I shall here give only the result. While hitherto in the best series of observations the probable error of a velocity derived from a plate amounted to not less than 0.25 km, the new method with the comparator succeeds in determining the motion within 0.1 km.

In conclusion I will remark that the comparator, beside its use in the measurement of line displacement, as described here, is also suitable for all comparative studies of various spectra, and for the investigation of the errors of small scales.

ASTROPHYSIKALISCHES OBSERVATORIUM,
Potsdam, June 1906

Reviews

Vorlesungen über theoretische Spektroskopie Von A. Garbasso. Leipzig: Barth, 1906. Pp. viii + 256, with 65 figures. Marks, 8.

This series of twenty lectures by Professor Garbasso represents one of the courses which he offers in the University of Genoa. The title alone raises a rather large question as to whether the science of spectroscopy is yet in a stage of advancement sufficient to permit of a general theory. In the opinion of many, including, we believe, Professor Garbasso himself, this question must be answered in the negative. The attempt, therefore, to see what can be done in the way of reaching a general view along different lines, such as the mechanical, electromagnetic, or electrostatic, is for this very reason exceedingly interesting.

It is important, however, for the reader to bear in mind what the author distinctly states at the beginning, namely that he has omitted from the volume everything which may not be considered as the description of a physically possible model.

The work is divided into four sections, of which the first is devoted to the experimental data of the science and to the mathematical tools employed in their investigation. The second, third, and fourth parts are given to the mechanical, electromagnetic, and electrostatic explanations, respectively.

The description of phenomena to be explained includes normal and anomalous dispersion, the Kayser-and-Runge series, variation of spectrum from one part of source to another, dependence of spectrum (of any one element) upon various physical conditions, phenomena of absorption, and the Zeeman effect.

The second lecture deals with an interesting attempt, on the part of Garbasso and Aschkinass, to build up a reflecting, refracting, and dispersing body from a large number of Hertzian oscillators. A short chapter on optical resonance, a clear statement concerning the use of the Lagrangian equations, and a few general mathematical theorems complete the first section of the volume.

The second section applies the equations of Lagrange, in a rather elegant manner, to the problem of normal dispersion according to the views of Cauchy, to the problem of anomalous dispersion according to

the mathematical theory of Helmholtz; and to the problem of the complex pendulum giving two frequencies and furnishing a mechanical analogue to illustrate Kundt's rule for the displacement of absorption bands in chemical compounds.

The third section is given over to various systems of Hertzian oscillators. Helmholtz's electromagnetic theory of dispersion is clearly set forth, Cauchy's formula coming out, of course, as a special case. The periods of various conductors and systems of conductors are determined with exceeding neatness by the Lagrangian method. Many of these conductors yield radiation of more than one frequency, thus giving groups of lines, which the author calls "doublets," "triplets," etc. But in no case is the approximation to Kayser and Runge's formula (i. e., all the frequencies in the series determined by three constants) sufficiently close to give one confidence of being on the right track. Thus, on p. 180, where a series of four lines is derived and discussed, it will be observed that three of these are used in the determination of the constants, so that only one line remains by which to test the formula. And while the agreement in this one case is fairly good, it does not follow that, in a series consisting of a larger number of lines, the agreement would be at all satisfactory.

In Part IV the electrodynamics of moving bodies is taken up, the Zeeman phenomenon is predicted and discussed very clearly—all in terms of the equations of Lagrange and Maxwell. Stoney's planetary (or kinematic) theory of the atom is set forth in a simple and clear manner.

The last two chapters contain an altogether excellent exposition of the atom proposed by Kelvin and profoundly modified by Lorentz, J. J. Thomson, and others.

The history of spectroscopy is filled with chapters which teach us to be very careful in the interpretation of experimental facts. Various interpretations have been given by different men at different times, to such a phenomenon as the long and short lines of Lockyer, or to the curvature of the lines in the Schuster-Hemsalech experiment, or to Lenard's separation of the electric arc into various regions by means of the slitless spectroscope. This variety of interpretation given to fundamental phenomena shows us at once how much we are in need of, and yet how far we are from, a general theory of spectroscopy.

The volume under review represents a laudable attempt to try out several different hypotheses. It will be of especial interest to the student of dynamics, containing as it does a series of illustrations of good dynamical method.

H. C.

A Compendium of Spherical Astronomy, with its Applications to the Determination and Reduction of Positions of the Fixed Stars. By SIMON NEWCOMB. New York: The Macmillan Co.; London: Macmillan & Co., Ltd., 1906. Pp. xviii+444, with 36 figures. $3, net.

Newcomb's *Compendium of Spherical Astronomy* is the most important addition to the literature of the subject since the appearance of the works of Chauvenet and Oppolzer. The volume is invaluable both to the advanced student and to the professional astronomer. It contains much already familiar, though presented with a freshness and directness that is new; a considerable amount of material hitherto inaccessible; and, most important of all, the results of the author's large experience with all that concerns the positions and proper motions of the stars. But it is evident from almost the first page that the discussion of the fundamental question of stellar positions and motions has been in the mind of the author as the real purpose of the book, for in the earlier chapters everything is prepared for the attack upon this problem, while time determination is barely touched upon, and the theory of the determination of latitude, eclipses, occultations, and transits is not even mentioned.

The space allotted to this review admits of only the briefest mention of the contents. The volume is divided into three parts. The first deals with preliminary subjects, including an introductory chapter of interest to the computer, a chapter on interpolation and mechanical quadratures, and one on the method of least squares. The last, of 44 pages, accepts the Gaussian formula of probability without development; and gives a concise statement of the different classes of errors and their determination, of the combination of observations by weight, and of the derivation and solution of a series of normal equations. Applications to cases of three and two unknowns are also made, that for the latter being arranged with special reference to the determination of proper motions.

Part II deals with the fundamental principles of spherical astronomy, which are developed with considerable detail. Spherical co-ordinates, the measure of time, parallax, aberration, astronomical refraction, and precession and nutation are the subjects treated. The last two, contained in chapters viii and ix, will be of the greatest interest to the professional astronomer. The chapter on refraction summarizes the results of recent meteorological investigations in their bearing upon the relation connecting atmospheric density and altitude, states the hypotheses of Newton, Bouguer, Bessel, and Ivory, and adds a fifth, which assumes a decrease in temperature in constant geometrical progression with the altitude. Two

forms are derived for the general equation for refraction. The first of these is the more useful when the zenith distance is not near 90°. It has the form

$$R = am \tan z,$$

where a is a constant, and

$$m = 1 - m_1 \sec^2 z + m_2 \sec^4 z - \ldots\ldots$$

The coefficient m_1 is shown to be independent of the relation connecting density with altitude. Whatever hypothesis is made, therefore, concerning the relation between these two quantities affects only the remaining terms of m, and since $m_2 < 0.000005$, and the coefficients of the higher terms are even smaller, it appears that the particular hypothesis used, excepting Bessel's, which deviates in another particular, is of importance only for large values of the zenith distance. This fact is of course well known, but the author's method of development exhibits the cause with especial clearness. The integrations necessary for the evaluation of m_2, m_3, etc., are carried through for the hypotheses of Newton and Ivory. The chapter ends with an account of the method employed in the construction of tables of refraction.

Chapter ix derives the numerical values of the various precessional motions and of the obliquity from the fundamental data for the motions of the poles of the equator and ecliptic, gives a clear exposition of the relation of nutation to precession, and contains a list of the usually included nutation terms.

Part III deals with the reduction and determination of the positions of the fixed stars. The first two chapters of this part form an important contribution to astronomical science. They contain improved methods for the reduction of star positions from one epoch to another and to apparent place. Rigorous trigonometrical developments are given for circumpolar stars as well as approximate formulæ for the more frequently occurring cases. The application of both methods of reduction is facilitated by the introduction of tables which are given in the Appendix. The processes here presented are designed to replace the older methods which, in certain cases, are unsatisfactory on account of the long interval through which precessional and proper motion reductions must now be made.

Chapter xii gives a clear account of the principles underlying the meridian determination of right ascensions and declinations, both fundamental and differential. The last chapter is devoted to an exposition of what the author has found to be the best method of deriving positions and proper motions from published results of observation. It is prefaced with a brief historical account of the development of meridian circle obser-

vation, and contains besides a description of the procedure involved in the formation of a fundamental system, with special references to the systems of Auwers, Boss, and Newcomb.

The notes and references appended to each chapter contain much valuable bibliographical material and many comments, suggestive and critical. The most important formulæ and the tables are collected in an appendix of 60 pages.

The usual number of misprints, apparently inevitable in a first edition, have made their appearance, but none of those noted are likely to cause the reader any great difficulty.

F. H. SEARES.

Elementi di Astronomia ad uso delle Scuole e per Istruzione privata. Compilati dal P. ADOLFO MÜLLER d. C. d. G. Roma: Desclée, Lefebvre E. C. Vol. I, Astrometria, Astromeccanica, 1904. Vol. II, Astrofisica, Astrochronaca, 1906. Pp. 600 each, with a total of 450 figures.

These two excellent volumes will meet with a hearty welcome from all who wish to be informed not only on the essentials of astronomy, but also on its connection with other sciences, its history, development, and present status.

Astrometry, in Vol. I, embraces the doctrine of the sphere, the construction of ancient and modern astronomical instruments, and some elementary mathematical formulæ in their application to the problems of practical astronomy. Astromechanics presents ancient and modern ideas regarding the mechanics of the solar system, eclipses, the calendar in its various transformations, the Galileo question, and the like.

Astrophysics, in Vol. II, contains short treatises on photography, spectroscopy, geology, meteorology, and terrestrial magnetism, and then investigates the physical constitution of the heavenly bodies. There is also a very condensed history of astronomy for handy reference.

Literary men will be pleased with the author's profuse references and quotations from all the great masters, from Aristotle, Plato, Cicero, Seneca, Ptolemy, Copernicus, Galileo, Newton, Herschel, and a host of others. The progress of the science is shown not only by the dicta of its great pioneers, but also by engravings of their instruments and the results of their observations. As the author is Father Secchi's legitimate successor, it was to be expected that, especially in astrophysics, Secchi's views should be prominently presented. The fact that the work was written in the Italian tongue and printed in Rome, will also appeal to some readers.

WILLIAM F. RIGGE.

NOTICE

The scope of the ASTROPHYSICAL JOURNAL includes all investigations of radiant energy, whether conducted in the observatory or in the laboratory. The subjects to which special attention will be given are photographic and visual observations of the heavenly bodies (other than those pertaining to "astronomy of position"); spectroscopic, photometric, bolometric, and radiometric work of all kinds; descriptions of instruments and apparatus used in such investigations; and theoretical papers bearing on any of these subjects.

In the department of *Minor Contributions and Notes* subjects may be discussed which belong to other closely related fields of investigation.

Articles written in any language will be accepted for publication, but unless a wish to the contrary is expressed by the author, they will be translated into English. Tables of wave-lengths will be printed with the short wave-lengths at the top, and maps of spectra with the red end on the right, unless the author requests that the reverse procedure be followed.

Authors are particularly requested to employ uniformly the metric units of length and mass; the English equivalents may be added if desired.

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